Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

P.D.D.C. Sem- I Remedial Examination March / April 2010

Subject code: X 10001 **Subject Name: Mathematics – I**

Date: 30 / 03 / 2010 **Time: 12.00 noon – 2.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed.

(a) Find the unit normal vector to the surface $x^2 + y^2 + z^2 = 7$ at (1, -1, 2). **02**

(b) Trace the curve $y^2(2a - x) = x^3$. **03**

(c) Solve $(x^2 + y^2 - a^2)xdx + (x^2 + y^2 - b^2)ydy = 0$. **03**

(d) Determine rank of the following matrix by row echelon form. **03**

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 6 & 8 & 10 \\ 7 & 10 & 13 & 16 \end{bmatrix}$$

(e) Find the inverse of the matrix **03**

$$A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$, using gauss-Jordan method.

Q.2 Attempt the following:

(a) Solve x + y +2z = 9 **03**

2x + 4y - 3z = 1

3x + 6y - 5z = 0 by Gaussian elimination and back substitution.

(b) Find the eigen values and eigen vectors of the matrix **04**

$$A=\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(c) Solve the following differential equations:

(i) $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$. **04**

(ii) $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0.$ **03**

OR

(c) Solve the following differential equations:

(i) $\frac{dy}{dx} + 2y\tan x = \sin x$ given that y = 0 when x = $\frac{\pi}{3}$ **04**

(ii) $\frac{dy}{dx} + \frac{1}{x}y = x^2y^6$ **03**

Q.3 Attempt the following:

(a) If v =log r, where $r^2 = x^2 + y^2$,show that **05**

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

(b) If u = $\tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$,show that **05**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u$

(c) If $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\frac{y}{x}$, evaluate $\frac{\partial(r,\theta)}{\partial(x,y)}$. **04**

OR

Q.3 Attempt the following:

(a) If u = log (tanx+tany+tanz), prove that **05**

$$\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 0$$

(b) Find the maximum and minimum values of **05**

$$x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$$

(c) The period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. **04**

If T is computed using l = 8 ft , g = 32 $ft/\sec^2$, find approximate error in T if true values are l = 8.05 ft and g = 32.01 $ft/\sec^2$.

Q.4 Attempt the following:

(a) Evaluate $\iint_R xydydx$ where R is the positive quadrant of the circle $x^2 + y^2 = a^2$. **05**

(b) change the order of integration in the integral $\int_0^{2a}\int_0^{\frac{x^2}{4a}} xydydx$ **05**

and hence evaluate it.

(c) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)dydxdz$. **04**

OR

Q.4 Attempt the following:

(a) Evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2 + y^2)dxdy$ by changing into polar co-ordinates **05**

(b) By double integration, find the area common to the curves $y^2 = 8x$ and $x^2 = 8y$. **05**

(c) Find the volume of the solid bounded by the surfaces x = 0,y = 0, z = 0 and x + y + z = 1. **04**

Q.5 Attempt the following:

(a) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at time t = 1 in the direction $\hat{i} + \hat{j} + 3\hat{k}$. **05**

(b) A vector field is given by $\overline{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that $\overline{F}$ is irrotational and find its scalar potential. **05**

(c) The current i flowing in the circuit containing resistance R, inductance L and e.m.f E Satisfies the differential equation $L\frac{di}{dt} + Ri = E$. Prove that **04**

$$i = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right), \text{ if } i = 0, \text{when } t = 0.$$

OR

Q.5 Attempt the following:

(a) Using the line integral, compute the work done by the force $\overline{F} = y\hat{i} + xz\hat{j} - x\hat{k}$ When it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the Curve $x = 2t^2, y = t, z = t^3$. **05**

(b) Verify Green's theorem in the plane for $\oint_c (2x - y^2)dx - xydy$, where c is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. **05**

(c) Find the orthogonal trajectories of the circles $(x - a)^2 + y^2 = a^2$. **04**

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