

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-III Remedial Examination May 2011

Subject code: 130001

Date: 31-05-2011

Subject Name: Mathematics-III

Time: 10.30 am – 01.30 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** Do as Directed
- i) Solve the Euler-Cauchy equation $x^2 y'' - 2.5xy' - 2.0y = 0$ 2
- ii) Find the Laplace transforms of $\left[\frac{\sin wt}{t} \right]$ 2
- iii) Solve $y' = 2xy$ by power series method. 2
- iv) Find the Fourier cosine and sine transforms of the function 2
- $$f(x) = \begin{cases} k & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$
- v) Write duplication formula. Use it to find the value of $\left[\frac{1}{4} \right] \left[\frac{3}{4} \right]$ 2
- vi) Write Abel-Liouville formula. Use it to check that the set 2
- $$\{x, x^2, x \log|x|\}$$
- is a basis for some third order linear ordinary differential equation.
- vii) Obtain $L^{-1} \left\{ \log \frac{1}{s} \right\}$ 2
- Q.2 (a)** i) Find the order and degree of the differential equation 1
- $$\left[\frac{dy}{dx} + y \right]^{\frac{1}{2}} = \sin x$$
- ii) To solve heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. How many initial and boundary 2
- conditions requires.
- iii) Evaluate $\int_{-1}^1 p_3(x)p_2(x)dx$ 2
- iv) Prove that $\frac{\beta(m+1, n)}{\beta(m, n)} = \frac{m}{m+n}$ 2

- (b) i) Solve $(D^2 + a^2)y = \cos ecax$ 4
 ii) Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$ 3
- OR
- (b) i) Solve $(D^4 + 2a^2D^2 + a^4)y = \cos ax$ 4
 ii) Obtain the second linearly independent solution of $xy'' + 2y' + xy = 0$ given that $y_1(x) = \frac{\sin x}{x}$ is one solution. 3
- Q.3** (a) Solve $x^3y''' + 2x^2y'' + 2y = 10\left(x + \frac{1}{x}\right)$ 4
 (b) Solve the initial value problem by method of undetermined coefficients $y''' + 3y'' + 3y' + y = 30e^{-x}$, $y(0) = 3$, $y'(0) = -3$, $y''(0) = -47$, 4
 (c) Solve the simultaneous equations: Using Laplace transform $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$ given $x(0) = 1$, $y(0) = 0$ 6
- OR
- Q.3** (a) Solve $x^2y'' - 4xy' + 6y = 21x^{-4}$ 4
 (b) Solve the nonhomogeneous Euler-Cauchy equation $x^3y''' - 3x^2y'' + 6xy' - 6y = x^4 \log x$ by Variation of parameters method. 4
 (c) i) Find the Laplace transform of the function $f(t) = |\sin wt|$, $t \geq 0$ 3
 ii) Using Convolution theorem, find the inverse Laplace transform of $\frac{1}{(s^2 + a^2)^2}$ 3
- Q.4** (a) Express $x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of Legendre's polynomials, by using Rodrigue's Formula. 4
 (b) i) Find the generalized Fourier series expansion of $f(x)$, $0 < x < 3$ arising from the eigenfunction of $y'' + \lambda y = 0$, $0 < x < l$; $y'(0) = 0$, $y'(l) = 0$ 4
 ii) Obtain the value of $J_{3/2}(x)$ 3
 (c) Show that $\int_{-1}^1 \frac{p_n(x)}{\sqrt{1-2xt+t^2}} dx = \frac{2}{2n+1} t^n$ 3
- OR
- Q.4** (a) Find Power series solution of the equation $(1-x^2)y'' - xy' + py = 0$, p is an arbitrary constant. 4
 (b) Find the series solution of $xy'' + y' + xy = 0$ 7
 (c) Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ about an ordinary point. 3

- Q.5 (a)** Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and use it to **4**

evaluate $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$

- (b)** Find the Fourier series for $f(x) = |\sin x|$ in $-\pi < x < \pi$ **4**

- (c)** If the string of length L is initially at rest in equilibrium position and each of its points is given the velocity. $u_0 \sin\left(\frac{3\pi x}{L}\right) \cos\left(\frac{2\pi x}{L}\right)$ **6**

Where $0 \leq x \leq L$ at $t = 0$, determine the displacement $u(x, t)$.

OR

- Q.5 (a)** Find half-range cosine series for $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ **4**

- (b)** Find Fourier series for the function $f(x)$ given by **4**

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}; & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}; & 0 \leq x \leq \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

- (c)** A rod 30 cm long has its end A and B kept $20^\circ C$ and $80^\circ C$ respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to $0^\circ C$ and kept so. Find the resulting temperature function $u(x, t)$ from the end A. **6**
