

**GUJARAT TECHNOLOGICAL UNIVERSITY****B. E. Sem - IV Examination - June- 2011****Subject code: 140001****Subject Name: Mathematics-4****Date: 02/06/2011****Time: 10.30 am – 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Use residues to evaluate  $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$  **05**
- (b) 1. Find real and imaginary part of  $(-1-i)^7 + (-1+i)^7$ . **03**  
 2. Check whether the following functions are analytic or not at any point: **03**  
 a.  $f(z) = e^{\bar{z}}$  b.  $f(z) = 2x + ixy^2$   
 3. Prove that  $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$  **03**
- Q.2** (a) 1. Evaluate  $\int_c f(z) dz$  where  $f(z)$  is defined by **03**  

$$f(z) = \begin{cases} 1 & \text{when } y < 0 \\ 4y & \text{when } y > 0 \end{cases}$$
 and  $c$  is the arc from  $z = -1 - i$  to  $z = 1 + i$  along the curve  $y = x^3$   
 2. Evaluate  $\int_c \frac{\cos \pi z^2}{(z-2)(z-1)} dz$ , where  $c$  is the circle  $|z| = 3$ . **03**
- (b) 1. Find the Laurent's expansion of **04**  

$$f(z) = \frac{7z-2}{(z+1)z(z-2)}$$
 in the region  $1 < z+1 < 3$ .  
 2. Find bilinear transformation which maps the points  $z = 1, i, -1$  onto the points  $w = i, 0, -i$ . Hence find the image of  $|z| < 1$ . **04**
- OR**
- (b) 1. Show that  $u(x, y) = 2x - x^3 + 3xy^2$  is harmonic in some domain and find a harmonic conjugate of  $u(x, y)$ . **04**  
 2. Find the image of the semi-infinite strip  $x > 0, 0 < y < 2$  when  $w = iz + 1$ . Sketch the strip and its image **04**
- Q.3** (a) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and residue at each pole. Hence evaluate  $\int_c f(z) dz$  where  $c$  is the circle  $|z| = 3$ . **05**
- (b) Express the function  $\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}$  as a sum of partial fraction, using Lagrange's formula. **05**
- (c) State Trapezoidal rule with  $n=10$  and evaluate  $\int_0^1 e^x dx$ . **04**
- OR**
- Q.3** (a) Evaluate  $\int_0^{\pi} \frac{1}{17-8 \cos \theta} d\theta$ , by integrating around a unit circle. **05**
- (b) From the following table, find  $f(x)$  using Newton's divided difference formula. **05**
- |        |   |   |   |   |
|--------|---|---|---|---|
| $x$    | 1 | 2 | 7 | 8 |
| $f(x)$ | 1 | 5 | 5 | 4 |
- (c) Evaluate  $\int_0^6 \frac{1}{1+x} dx$  taking  $h=1$  using Simpson's 1/3 rule. Hence, obtain an approximate value of  $\log_e 7$ . **04**

- Q.4 (a)** Employ Stirling's formula to compute  $y(35)$  from the following table. **05**

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| $x$ | 20  | 30  | 40  | 50  |
| $y$ | 512 | 439 | 346 | 243 |

- (b)** With the usual notation, show that **05**

(1).  $\Delta = 1 - e^{hD}$  (2).  $(1 + \Delta)(1 - \nabla) = 1$ .

- (C)** Solve:  $x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40$  by Gauss elimination method. **04**

**OR**

- Q.4 (a)** Solve the following system of equation by using Gauss-Seidel method correct up to two decimal places. **05**

$$20x + 2y + z = 30, x - 40y + 3z = -75,$$

$$2x - y + 10z = 30.$$

- (b)** Using Newton's forward formula, find the value of  $f(1.6)$  if **05**

|        |      |      |      |     |
|--------|------|------|------|-----|
| $x$    | 1    | 1.4  | 1.8  | 2.2 |
| $f(x)$ | 3.49 | 4.82 | 5.96 | 6.5 |

- (c)** Use power method to find the largest eigen value of the matrix **04**

$$A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}.$$

- Q.5 (a)** Find a root of the equation  $x^3 - 4x - 9 = 0$ , using the bisection method in four stages. **05**

- (b)** Using Taylor series method, find correct to four decimal places, the value of  $y(0.1)$ , given  $\frac{dy}{dx} = x^2 + y^2$  and  $y(0) = 1$ . **05**

- (c)** Find the positive root of  $x = \cos x$  using Newton's method correct to 3 decimal places. **04**

**OR**

- Q.5 (a)** Apply the fourth order Rung-Kutta method to find  $y(0.2)$ , given  $\frac{dy}{dx} = x + y, y(0) = 1$ . (Take  $h=0.1$ ) **05**

- (b)** Develop a recurrence formula for finding  $\sqrt{N}$ , using Newton-Raphson method and hence compute  $\sqrt{27}$  to three decimal places. **05**

- (c)** Using Euler's method, Find  $y(0.2)$  given  $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$ . (Take  $h=0.1$ ) **04**

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