

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE SEM-IV Examination-Nov/Dec-2011****Subject code: 140001****Date: 21/11/2011****Subject Name: Mathematics-4****Time: 02.30 pm -5.30 pm****Total marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1**

Do as Directed:

**14**

- (i) Show that  $f(z) = z \operatorname{Im}(z)$  is differentiable only at  $z = 0$  and  $f'(0) = 0$ .
- (ii) Find the principal argument of  $z = \frac{-2}{1+i\sqrt{3}}$ .
- (iii) Find and plot all the roots of  $(1+i)^{\frac{1}{3}}$ .
- (iv) Show that the set of values of  $\log(i^2)$  is not the same as the set of values  $2 \log i$ .
- (v) Find the maclurin series representation of  $f(z) = \sin z$  in the region  $|z| < \infty$ .
- (vi) Evaluate  $\int_0^{4+2i} z dz$  along the curve  $z = t^2 + it$ .
- (vii) Find the residue at  $z = 0$  of  $f(z) = z \cos \frac{1}{z}$ .

**Q.2 (a)**

- (i) Evaluate  $\int_0^1 \frac{dx}{1+x^2}$ , using trapezoidal rule with  $h = 0.2$ .

**03**

- (ii) Compute  $f(8)$  from the following values using Newton's Divided difference formula.

**04**

|        |    |     |     |     |      |      |
|--------|----|-----|-----|-----|------|------|
| $x$    | 4  | 5   | 7   | 10  | 11   | 13   |
| $f(x)$ | 48 | 100 | 244 | 900 | 1210 | 2028 |

**(b)**

- (i) Perform the five iterations of the bisection method to Obtain a root of the equation  $f(x) = x^3 - x - 1 = 0$ .
- (ii) Solve the following system of equations by Gauss-Seidel Method correct to three decimal places.

**03****04**

$$2x + y + 54z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

**OR**

- (b) (i) Solve the following system of equation using Gauss Elimination method with partial Pivoting. 04
- $$\begin{aligned}x + y + z &= 7 \\3x + 3y + 4z &= 24 \\2x + y + 3z &= 16\end{aligned}$$
- (ii) Find the dominant eigen value of  $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$  by power Method. 03

- Q.3 (a)** Define mobius transformation .Also find the image of the circle  $|z|=1$  in the w-plane under the mobius transformation  $\omega = f(z) = \frac{z-i}{1-iz}$ . Also find the fixed points of  $f$ . 05
- (b) Attempt the following: 09

(i) State Cauchy's integral formula and hence evaluate  $\oint_C \frac{\sin 3z}{z + \frac{\pi}{2}} dz$

where C is the circle  $|z|=5$ .

(ii) Show that  $u(x, y) = e^{x^2-y^2} \cos(2xy)$  is harmonic everywhere and find a conjugate harmonic for  $u(x, y)$ .

(iii) Expand  $f(z) = \frac{1}{(z+1)(z+3)}$  in Laurent's series in the interval  $1 < |z| < 3$ .

**OR**

- Q.3 (a)** Evaluate  $\oint_C \frac{dz}{\sinh 2z}$ , where  $C : |z|=2$ . 05
- (b) (i) Using the residue theorem, evaluate  $\int_0^{2\pi} \frac{4d\theta}{5+4\sin\theta}$ . 09
- (ii) Prove that all the roots of  $z^7 - 5z^3 + 12 = 0$  lie between the Circles  $|z|=1$  and  $|z|=2$ , using Rouche's theorem.
- (iii) Find the principal value of  $\left[ \frac{e}{2} (-1-i\sqrt{3}) \right]^{3\pi i}$ .

- Q.4 (a)** (i) Using Lagrange's formula to fit a polynomial to the data 04

|   |    |   |   |    |
|---|----|---|---|----|
| x | -1 | 0 | 2 | 3  |
| y | 8  | 3 | 1 | 12 |

And hence find  $y(x=2)$ .

- (ii) A river is 80 meters wide. The depth 'd' in meters at a distance x meters from one bank is given by the following table calculate the area of cross-section of the river using Simpson's  $\frac{1}{3}$  rule 05

|   |   |    |    |    |    |    |    |    |    |
|---|---|----|----|----|----|----|----|----|----|
| x | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| d | 0 | 4  | 7  | 9  | 12 | 15 | 14 | 8  | 3  |

- (b) Using improved Euler's method, solve  $\frac{dy}{dx} = 1 - y$  with the initial condition  $y(0) = 0$  and tabulate the solutions at  $x = 0.1, 0.2$ . Compare the answer with exact solution. 05

OR

- Q.4 (a) Evaluate  $\int_0^1 \frac{dt}{1+t}$  by the Gaussian formula with one point, two point and three points. 05

- (b) (i) Find the value of  $\sin 52^\circ$  from the following table 05

| $\theta^\circ$      | $45^\circ$ | $50^\circ$ | $55^\circ$ | $60^\circ$ |
|---------------------|------------|------------|------------|------------|
| $\sin \theta^\circ$ | 0.7071     | 0.7660     | 0.8192     | 0.8660     |

- (ii) Find to four decimal places, the smallest root of the equation  $\sin x = e^{-x}$  using the Newton-Raphson starting With  $x_0 = 0.6$ . 04

- Q.5 (a) (i) Given:  $10 \frac{dy}{dx} = x^2 + y^2, y(0) = 1$ . Using fourth order Runge-Kutta method. Find  $y$  05

(0.2) with  $h=0.1$ . (ii) Use Euler's method to obtain an approximate value of  $y(0.4)$

For the equation  $y' = x + y, y(0) = 1$  with  $h=0.1$ . 04

- (b) Evaluate the integral  $\int_4^{5.2} \log_e x dx$  using Simpson's  $\frac{3}{8}$  rule. 05

OR

- Q.5 (a) (i) Find the image of the region bounded by  $1 \leq r \leq 2$  and  $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$  in the  $z$ - 05

plane under the transformation  $w = z^2$ . Show the regions graphically.

(ii) Check whether the function  $f(z) = \sin z$  is analytic or not. If analytic, find its derivative. 04

- (b) Evaluate  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$  where  $C$  is the circle  $|z| = 3$ . 05

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