Seat No.: _________ Enrolment No._______________

GUJARAT TECHNOLOGICAL UNIVERSITY
B.E SEM-VII Examination-Nov/Dec.-2011

Subject code: 172007 **Date: 29/11/2011**
Subject Name: Modern Control Systems
Time: 10.30 am-01.00 pm **Total marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Consider a Unity-feedback type-1 system with open loop transfer function **07**

$$G(s) = \frac{k}{s(s+2)}$$

It is desired to designed a cascade lead compensator such that the dominant closed-loop poles provide a damping ratio $\xi = 0.5$ and have undamped natural frequency $\omega n = 4 rad/s$.

(b) The Uncompensated system transfer function is given by **07**

$$G(s) = \frac{k}{s(s+2)}$$

The system is to be compensated to meet the following specification.
Damping ratio, $\xi = 0.707$
Settling Time, $t_s \le 5 \text{ sec}$
Velocity error constant, $k_v \ge 4$
Design a suitable lag compensator for the desired specification.

Q.2 **(a)** Consider a unity-feedback control system with open-loop transfer function **07**

$$G(s) = \frac{k}{s^2(0.2s+1)}$$

Assume that the System is required to be compensated to meet the following specifications:
Acceleration error constant $ka = 10$
Phase Margin $\ge 35^\circ$
Design a suitable lead compensator for the uncompensated system.

(b) Consider the unity feedback system whose open-loop transfer function is **07**

$$G(s) = \frac{k}{s(s+1)}$$

The system is to be compensated to meet the following specifications:
Velocity error constant $kv = 10$.
Phase Margin $\phi M \ge 45^0$
Design a suitable compensator for the uncompensated system.

OR

(b) Consider a plant with transfer function **07**

$$G(s) = \frac{4}{s(s+0.5)}$$

It is desired to meet the following specification:

Damping ratio of dominant closed-loop poles, $\xi = 0.5$

Undamped natural frequency of dominant closed-loop poles,

$\omega n = 5 rad/\sec$

Velocity error constant=80 $\sec^{-1}$

Design a suitable lag-lead compensator for the uncompensated root loci.

Q.3 **(a)** A system is represented by the equations given below **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Given x(0)=[1 1]^T

Determine (1) state transition matrix $\phi(t)$

(2) Zero Input response

(3) Zero state response for u=1

(4) Total Response.

(b) Represent the following differential equation in state variable form **07**

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u(t)$$

Draw the state diagram and find the state transition matrix.

OR

Q.3 **(a)** Use Laplace transform method to determine the time response given that **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{pmatrix} -3 & 1 \\ -4 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$X_1(0)=0, X_2(0)=0$

And y=[1 1] $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(b) Check the controllability and Observability of the system having the following coefficient matrices: **07**

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{pmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, c^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$$

Q.4 **(a)** Define the definition of z-transform and find the z transform of the following function: **07**

(i) $f(t) = t e^{-t}$

(ii $f(t) = e^{-at} \sin wt$

(iii) $f(t) = u(t) - e^{-2t}$

(b) Explain the stability analysis of sampled-data control system and mapping from-plane to z-plane. **07**

OR

Q.4 **(a)** Find the inverse z-transform for the following function. **07**

1. $F(z) = \dfrac{0.632z}{z^2 - 1.368z + 0.368}$

2. $F(z) = \dfrac{z}{z^2 - z + 0.5}$

(b) Determine the stability of a sampled data control system having following characteristic polynomial using Jury's stability test. **07**

$$2z^4 + 8z^3 + 12z^2 + 5z + 1 = 0$$

Q.5 **(a)** A regulator system has the plant **07**

$$\dot{x} = Ax + Bu$$

$$y = cx$$

With $A = \begin{pmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{pmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$,c=[0 0 1]

Compute k so that the control law u= -kx places the closed-loop poles at $-2 \pm j3.464$,-5.

(b) Explain the pole placement design using state feedback. **07**

OR

Q.5 **(a)** Explain the aspects that must be taken into account while design the control system. **07**

(b) The transfer function of a control system is given by **07**

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 3s + 4}{s^3 + 2s^2 + 3s + 2}$$

Obtain a state model.

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