GUJARAT TECHNOLOGICAL UNIVERSITY
BE- Ist /IInd SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 110008   **Date: 06/06/2012**

Subject Name: Maths-I

Time: 10:30 am – 01:30 pm   **Total Marks: 70**

Instructions:
1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks**

Q.1   **(a)**   Evaluate : (i) $\lim\limits_{x \to 0} \left(\dfrac{1}{x}\right)^{1-\cos x}$

                 (ii) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^2 \tan x}$.   **04**

   (b)   Find the directional derivative of $f = 4xz^3 - 3x^2y^2z$ at (2,-1,2)   **03**
in the direction of $2\vec{i} + 3\vec{j} + 6\vec{k}$

   (c)   Find the Taylor series generated by $f(x) = \dfrac{1}{x}$ at a=2. Where ,if   **03**
anywhere , does the series converge to $\dfrac{1}{x}$?

   (d)   Find Maclaurin series for the function $x^4 - 2x^3 - 5x + 4$.   **02**

   (e)   Evaluate $\int\limits_0^1 \int\limits_0^{1-y} \int\limits_0^2 dx\,dy\,dz$.   **02**

Q.2   **(a)**   Discuss the convergence of the following series: **06**

     (i) $\sum\limits_{n=1}^{\infty} \dfrac{2n+1}{n^2+n+1}$ (ii) $\sum\limits_{n=1}^{\infty} \dfrac{n^2}{2^n}$ (iii) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n x^{n+1}}{2n-1}$

   (b)   State Direct comparison test for improper integrals. Using it show   **04**
that, $\int\limits_1^{\infty} e^{-x^2} dx$ is convergent.

   (c)   Evaluate the following improper integrals: **04**

     (i) $\int\limits_0^{\infty} \dfrac{dx}{x^2+1}$ (ii) $\int\limits_0^{\infty} \dfrac{dv}{(1+v^2)(1+\tan^{-1}v)}$

Q.3   **(a)**   (i) Show that $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2}, & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$ is continuous   **05**
at every point except at the origin.

     (ii) Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$,

     $x = \dfrac{r}{s}, y = r^2 + \ln s, z = 2r$.

   (b)   State Euler's theorem on homogeneous function of two variable and   **05**

apply it to evaluate $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ if $u = \tan^{-1}\left(\dfrac{x^2 + y^2}{x - y}\right)$.

(c) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$ prove that $\qquad$ **04**

$$\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y}\right)^2 u = \dfrac{-4}{(x + y)^2}.$$

Q.4

(a) Find the extreme values of $\sin x + \sin y + \sin(x + y)$, $(0 \le x, y \le \dfrac{\pi}{2})$. **05**

(b) Find the maximum and minimum values of the function **05**
$f(x, y) = 3x + 4y$ on the circle $(x^2 + y^2) = 1$ using the method of
Lagrange multipliers.

(c) Evaluate $\displaystyle\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \dfrac{2x - y}{2}\, dxdy$ by applying the transformation **04**

$u = \dfrac{2x - y}{2}, v = \dfrac{y}{2}$ and integrating over an appropriate region tn the
uv-plane.

Q.5 (a) Evaluate $\displaystyle\iint_R (y - 2x^2)dA$, Where R is the region bounded by the **05**

square $|x| + |y| = 1$.

(b) Sketch the region of integration. Change the order of integration and **05**

evaluate the integral $\displaystyle\int_0^1 \int_y^{\sqrt{y}} dxdy$.

(c) Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$. **04**

Q. 6 (a) Find the volume of the region that lies under the paraboloid **05**
$z = x^2 + y^2$ and above the triangle enclosed by the lines
$y = x$, $x = 0$, and $x + y = 2$ in the xy-plane.

(b) Find the area of the region R enclosed by the parabola $y = x^2$ and **05**
the line $y = x + 2$.

(c) Determine whether the vector field $\vec{u} = y^2\vec{i} + 2xy\vec{j} - z^2\vec{k}$ is **04**
solenoidal or irrotational at a point $(1,2,1)$?

Q.7 (a) State Green's theorem and using it, evaluate $\displaystyle\oint_c xydy - y^2 dx$, **05**

Where C is the square cut from the first quadrant by the lines $x = 1$
and $y = 1$.

(b) Verify stoke's theorem for $\vec{F} = xy^2\vec{i} + y\vec{j} + z^2 x\vec{k}$ for the surface of a **05**
rectangular lamina bounded by $x = 0, y = 0, x = 1, y = 2, z = 0$

(c) Evaluate $\displaystyle\int_c \vec{F}.d\vec{r}$, where $\vec{F} = y^2\vec{i} + 2xy\vec{j}$ and (i) C is the straight line **04**

from (0,0) to (1,2). (ii) C is the parabola $y = 2x^2$ from (0,0) to (1,2)

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