Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEM-III Examination May 2012

Subject code: 130001

Subject Name: Mathematics - III

Date: 14/05/2012 **Time: 02.30 pm – 05.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt all quations: **04**

(1) Solve the differential equation $xy\frac{dy}{dx} = 1 + x + y + xy$

(2) Find the general solution of $\frac{d^4y}{dx^4} - 18\frac{d^2y}{dx^2} + 81y = 0$

(3) Find particular solution of $y = \frac{1}{(D+1)^2}\cosh x$, where $D = \frac{d}{dx}$

(4) Find the value of $\Gamma\frac{1}{4}\Gamma\frac{3}{4}$

(b) Attempt the following equations: **10**

(1) Determine the singular points of differential equation $2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$ and classify them as regular or irregular.

(2) Find half range cosine series for $f(x) = e^x$ in $(0,1)$.

(3) Find the fourier sine transform of $f(x) = e^{-2x} + e^{-3x}, x > 0$.

(4) Solve : $(x+y)^2\left[x\frac{dy}{dx} + y\right] = xy\left[1 + \frac{dy}{dx}\right]$ Evaluate :

(5) $\int_0^1 x^4 \cos^{-1} x dx$

Q.2 **(a)** Attempt the following quations:

(1) Find the Laplace transform of $f(t) = t^2 \sinh at$ **02**

(2) Find the Laplace transform of $f(t) = \begin{cases} 0, 0 < t < \Pi \\ \sin t, t > \Pi \end{cases}$ **02**

(3) Find the inverse Laplace transform of $\frac{5s+3}{(s^2+2s+5)(s-1)}$ **03**

(b) Attempt the following quations :

(1) Solve the differential equation : $(x^2y^2+2)ydx + (2 - x^2y^2)xdy = 0$. **03**

(2) Find the solution of differential equation $y'' - 5y' + 6y = 0$ with initial condition $y(1) = e^2$ and $y'(1) = 3e^2$. **02**

(3) Find the Laplace transform of $\frac{(1-\cos t)}{t}$ **02**

OR

(b) Attempt the following quations:

(1) Using Laplace transform solve the differential equation **03**

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t \quad \text{where } x(0)=0 \text{ and } x'(0)=1.$$

(2) Find the series solution of $(1+x^2)y''+xy'-9y=0$. **04**

Q.3 **(a)** Attempt the following quations

(1) Solve: $\frac{d^4y}{dt^4}-2\frac{d^2y}{dt^2}+y=\cos t+e^{2t}+e^t$ **03**

(2) Solve: $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=\frac{e^{2x}}{x^5}$ **03**

(3) The Bessel equation of of order zero is $x^2y''+xy'+x^2y=0$ then **04**

(i) find the roots of the indicial equation

(ii) show that one solution for $x>0$ is $y=c_0J_0(x)$

where, $J_0(x)=1+\sum\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$

(b) Find fourier series for $f(x)=\begin{cases}-\Pi, -\Pi\le x\le 0\\ x, 0\le x\le \Pi\end{cases}$ **04**

and show that $\frac{\Pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots\ldots$

OR

Q.3 **(a)** Attempt the following quations

(1) Solve: $\frac{d^2y}{dx^3}-\frac{d^2y}{dx^2}+\frac{3dy}{dx}+5y=e^x\cos 3x$ **03**

(2) Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos(\log(1+x)\ )$ **03**

(3) Find the series solution using by Fobenius method **04**

$xy''+y'-y=0$

(b) Find fourier series for $f(x)=2x-x^2$ in the interval $(0,3)$. **04**

Q.4 **(a)** Attempt the following quations :

(1) Solve the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2e^x\cos\left(x/2\right)$ Solve the **03**

(2) differential equation $(X^2D^2-3XD+4)y=x^2$ given that **03**

$y(1)=1$ and $y'(1)=0$.

(3) Evaluate : $\int_3^7 (x-3)^{1/4}(7-x)^{1/4}dx$ **02**

(b) Attempt the following quations:

(1) Prove that in usual notation **03**

$4J_n''(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x)$

(2) Find Laplace transform of (i) $e^{-3t}u(t-2)$, (ii) $\int_0^t e^{-u}\cos u\,du$ **03**

OR

Q.4 **(a)** Attempt the following quations:

(1) Solve the differential equation $\frac{d^3y}{dx^3}+\frac{dy}{dx}=\cos ecx$ by method of variation of parameters. **03**

(2) Solve : $\left(D^2-4D+4\right)y=\frac{e^{2x}}{1+x^2}$ where $D=\frac{d}{dx}$ **03**

(3) Evaluate : $\int_0^1 \left(x\log x\right)^3 dx$ **02**

(b) Attempt the following equation:

(1) Solve the differential equation $\frac{d^2y}{dt^2}+4y=f(t), y(0)=0, y'(0)=1$ by laplace transform **03**

where (i) $f(t)=\begin{cases}1, 0<t<1\\ 0, t>1\end{cases}$ (ii) $f(t)=H(t-2)$

(2) Find the fourier transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{\lambda^2}{2}}$ **03**

Q.5 **(a)** Attempt the following equation: **05**

(1) Find half Range cosine series for sinx in (0,Π) and show that

$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.........=\frac{\Pi}{4}$ And using parseval's Identity prove that

$\frac{1}{1^2.3^2}+\frac{1}{3^2.5^2}+\frac{1}{5^2.7^2}+.........=\frac{\Pi^2-8}{16}$

(2) Solve $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ by the method of separation of variables **04**

(b) A tightly stretched string with fixed end points x = 0 and x = L is initially Given the displacement $y=y_0\sin^3\left(\frac{\Pi x}{L}\right)$ If it is released from rest from this position then find the displacement y **05**

use the equation $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$

OR

Q.5 **(a)** Attempt the following equation:

(1) If $f(x)=\begin{cases}mx, 0\le x\le\frac{\Pi}{2}\\ m(\Pi-x), \frac{\Pi}{2}\le x\le\Pi\end{cases}$ then show that **05**

$f(x)=\frac{4m}{\Pi}\left\{\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}-..........\right\}$

(2) Determine the solution of one dimensional heat equation **04**

$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ where the boundary condition

are $u(0,t)=u(L,t)=0, t>0$ and the initial condition is

$u(x,0) = x,$, L being the length. $(0 < x < L)$

(b) Solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for the condition of heat along a rod without **05**

radiation subject to the condition (i) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = t$

(ii) $u = lx - x^2$ at $t = 0$ and for all x

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