

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- IVth SEMESTER-EXAMINATION – MAY/JUNE- 2012****Subject code: 140001****Date: 18/05/2012****Subject Name: Mathematics-IV****Time: 10:30 am – 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (1) Is $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$? Justify. 02**(2) Classify the poles of $f(z) = \frac{1}{z^2 - z^6}$. 02****(3) Evaluate $\int_C \frac{dz}{(z^2 + 1)^2}$ where $C : |z + i| = 1$. 03****(b) (1) Let $a > b > 0$. Prove that 04**

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a^2 - b^2} \left(\frac{e^{-b}}{b} - \frac{e^{-a}}{a} \right).$$

(2) Define fixed point of bilinear transformation. Find fixed point of $w = \frac{z+1}{z}$. Verify your result. 02**(3) Sketch $S = \{ z / -1 < \text{Im}(z) < 2 \}$. Is it Connected? 01****Q.2 (a) (1) Define Domain. Is the set $|z - 1 + 2i| \leq 2$ domain? 02****(2) Define $\text{Log}(x + iy)$. Determine $\text{Log}(1 - i)$. 02****(3) Discuss the rate of convergence of Newton Raphson's method. 03****(b) (1) Prove $\lim_{z \rightarrow 1} \frac{i \bar{z}}{3} = \frac{i}{3}$ by definition. 03****(2) Verify that C – R equations are satisfied at $z = 0$ for function 04**

$$f(z) = \begin{cases} \frac{-2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

OR**(b) (1) Suppose that $f(z)$ and $\overline{f(z)}$ both are analytic in a domain D . Prove that $f(z) = \text{constant}$. 04****(2) Define Harmonic function. Show that $u = x \sin x \cosh y - y \cos x \sinh y$ is harmonic. 03****Q.3 (a) (1) Evaluate $\int_C \bar{z} dz$ from $z = 1 - i$ to $z = 3 + 2i$ along the straight line. 05****(2) Determine the bilinear transformation which mapping the points $0, \infty, i$ in to $\infty, 1, 0$. 02****(b) (1) Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the region (i) $|z| < 2$ 05****(ii) $2 < |z| < 4$ (iii) $|z| > 4$** **(2) Define the isolated singular point and give an example. 02****OR**

- Q.3 (a)** (1) Evaluate $\int_C (x^2 - i y^2) dz$ along the parabola $y = 2x^2$ from $(1, 2)$ to $(2, 8)$. **04**
- (2) Determine and sketch the image of $|z| = 1$ under the transformation $w = z + i$. **03**
- (b)** (1) Find the series of $f(z) = \frac{z}{(z-1)(z-4)}$ in terms of $(z+3)$ valid for $|z+3| < 4$. **05**
- (2) Find the residue of $f(z) = \frac{1-e^z}{z^3}$ at $z = 0$. **02**
- Q.4 (a)** (1) Find the negative root of $x^3 - 7x + 3 = 0$ by bisection method up to three decimal places. **04**
- (2) Describe Euler's Method for first order ordinary differential equation. **03**
- (b)** (1) Determine the largest eigen values of $A = \begin{bmatrix} -1 & 1 & 4 \\ 10 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by power method. **04**
- (2) Obtain the Newton – Raphson formula from Taylor's Theorem. **03**
- OR**
- Q.4 (a)** (1) Let $f(40)=836$, $f(50)=682$, $f(60)=436$, $f(70)=272$. Use stirling's formula to find $f(55)$. **04**
- (2) Find a root of $x^4 - x^3 + 10x + 7 = 0$ correct to three decimal places between $a = -2$ and $b = -1$ by Newton – Raphson Method. **03**
- Q.4 (b)** (1) Derive Secant Method and solve $xe^x - 1 = 0$ correct to three decimal places between 0 and 1. **04**
- (2) Write $f(x) = x^4 - 2x^3 + x^2 - 2x + 1$ in factorial notation and find $\Delta^4 f(x)$. **03**
- Q.5 (a)** (1) Determine $y(0.1)$ and $y(0.2)$ correct to four decimal places from $\frac{dy}{dx} = 2x + y$, $y(0) = 1$. **04**
- Use fourth order Runge-Kutta method.
- (2) Determine the polynomial by Newton's forward difference formula from the following table. **03**
- | | | | | | | |
|-----|-----|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | -10 | -8 | -8 | -4 | 10 | 40 |
- (b)** (1) Consider following tabular values. **03**
- | | | | | | |
|-----|-----|-----|-----|-----|------|
| x | 50 | 100 | 150 | 200 | 250 |
| y | 618 | 724 | 805 | 906 | 1032 |
- Determine $y(300)$.
- (2) Consider following tabular values. **04**
- | | | | | | | | |
|------------|-------|-------|-------|-------|-------|-------|-------|
| x | 25.0 | 25.1 | 25.2 | 25.3 | 25.4 | 25.5 | 25.6 |
| $y = f(x)$ | 3.205 | 3.217 | 3.232 | 3.245 | 3.256 | 3.268 | 3.280 |
- Determine the area bounded by the given curve and X – axis between $x=25$ to $x=25.6$ by Trapezoidal rule and Weddle's rule.

OR

Q.5 (a) (1) Consider the following values. 04

x	10	11	12	13	14	15	16
y	1.02	0.94	0.89	0.79	0.71	0.62	0.55

Find $\int_{10}^{16} y dx$ by Simpson's $\frac{1}{3}$ rule and Weddle's rule.

(2) Solve $\frac{dy}{dx} = 3 + 2xy$ where $y(0) = 1$ for $x = 0.1$ by Picard's method. 03

(b) (1) Determine $y(12)$ by Lagrange Interpolation from following values. 03

x	11	13	14	18	20	23
y	25	47	68	82	102	124

(2) Solve the following system of linear equations by Gauss Jordan Method. 04

$$2x + 5y - 3z = 1, \quad 5x + y + 4z = 2, \quad 7x + 3y + z = 4$$
