

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III • EXAMINATION – SUMMER 2013****Subject Code: 130002****Date: 01-06-2013****Subject Name: Advanced Engineering Mathematics****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a) (i)** Solve $(1+y^2)\frac{dx}{dy} = \tan^{-1} y - x$. **03**
- (ii)** Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$. **04**
- (b)** Find power series solution of the of the equation **07**
 $\frac{d^2y}{dx^2} + xy = 0$ in powers of x .
- Q.2 (a)** Solve $x \frac{\partial u}{\partial x} - 2y \frac{\partial u}{\partial y} = 0$ using method of separation of variables. **07**
- (b)** Find the power series solution of the equation **07**
 $(x^2 + 1)y'' + xy' - xy = 0$ about $x = 0$.
- OR**
- (b)** Solve $(D^3 - D^2 - 6D)y = x^2 + 1$ **07**
- Q.3 (a)** Find the fourier series of the periodic function $f(x)$ with period 2π , **07**
defined as follows:

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x \leq 0 \\ x, & \text{for } 0 \leq x < \pi \end{cases}$$
- (b)** Find the fourier cosine series for $f(x) = x^2$, $0 < x \leq c$. Also sketch **07**
 $f(x)$.
- OR**
- Q.3 (a)** Find the fourier series of periodic function with period 2, which is **07**
given below.

$$f(x) = \begin{cases} 0 & -1 \leq x \leq 0 \\ x & 0 \leq x \leq 1 \end{cases}$$
- (b)** Find the fourier series of the periodic function with period 2 of **07**

$$f(x) = \begin{cases} \pi & 0 \leq x \leq 1 \\ \pi(2-x) & 1 \leq x \leq 2 \end{cases}$$
- Q.4 (a) (i)** Prove that $L(e^{-at}) = \frac{1}{s+a}$, $s > -a$ **03**
- (ii)** Prove that $L(t^n) = \frac{n!}{s^{n+1}}$, n being positive integer. **04**
- (b) (i)** Find $L^{-1}\left\{\log \frac{s+a}{s+b}\right\}$ **04**

- (ii) Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\}$ 03

OR

- Q.4 (a)** If $\overline{f(s)}$ is the laplace transform of $f(t)$ and $a \geq 0$, then 07
 prove that $L[f(t-a)u(t-a)] = e^{-as} \overline{f(s)}$

- (b) (i) Find the laplace transform $L\left[\int_0^t e^{-x} \cos x \, dx\right]$ 03

- (ii) Find the laplace transform $L\left[\int_0^t \int_0^t \sin au \, du \, du\right]$ 04

- Q.5 (a)** (i) Define Beta function and Rectangle function. 04

- (ii) Form partial differential equation $z = (x-2)^2 + (y-3)^2$ 03

- (b) Find the fourier transform of f defined by $f(t) = e^{-|t|}$. 07
 Sketch the graph.

OR

- Q.5 (a)** (i) Eliminate the function f from the relation $f(xy + z^2, x + y + z) = 0$. 04

- (ii) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$, and 03

$z = 0$, when y is an odd multiple of $\frac{\pi}{2}$.

- (b) (i) Solve $x(y-z)p + y(z-x)q = z(x-y)$ 04

- (ii) Solve $z = px + qy + p^2 q^2$ 03
