

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III • EXAMINATION – SUMMER • 2014****Subject Code: 130001****Date: 02-06-2014****Subject Name: Mathematics-III****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** (i) Find the general solution of the differential equation $y' = e^{2x+3y}$. **02**
 (ii) Find the particular solution of the differential equation $y'' + 4y = 2 \sin 3x$ by using method of undetermined coefficients. **02**

- (iii) Find the inverse Laplace transform of following function: **03**

$$\frac{s}{s^2 - 3s + 2}$$

- (b)** (i) Define Ordinary Point of the differential equation $y'' + P(x)y' + Q(x)y = 0$. **02**

- (ii) Find the value of $\left| \frac{3}{4} \right| \frac{1}{4}$. **02**

- (iii) Express the function $f(x) = x$ as a Fourier series in interval $[-\pi, \pi]$. **03**

- Q.2 (a)** (i) Evaluate: $\int_0^{\infty} x^2 e^{-x^4} dx$. **02**

- (ii) Solve: $(D^4 - 1)y = 0$. **02**

- (iii) Solve the partial differential equation: $u_{xy} = x^3 + y^3$. **03**

- (b)** (i) Find the Laplace transforms of function $f(t) = t^5 + \cos 5t + e^{-100t}$ **03**

- (ii) Using method of variation of parameters solve the differential equation $y'' + 4y = \tan 2x$. **04**

OR

- (b)** (i) Find the Laplace transforms of the function $f(t) = t \cosh t$. **03**

- (ii) Solve: $(D^2 - 25)y = \cos 5x$ **04**

- Q.3 (a)** Find the Laplace transforms of following functions: **07**

- (i) $\cos^3 t$ (ii) $\sin^2 t$.

- (b)** State Convolution Theorem and using it find inverse Laplace transform of **07**

$$\text{function } f(t) = \frac{s^2}{(s^2 + 4)(s^2 + 9)}$$

OR

- Q.3 (a)** Using Laplace transform solve the differential equation: **07**

$$y'' + 5y' + 6y = e^{-t}, y(0) = 0, y'(0) = -1.$$

- (b) Evaluate: (i) $\int_{-1}^1 (1-x^2)^n dx$, where n is a positive integer. 07
- (ii) $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} d\theta$.
- Q.4 (a)** (i) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right]$ 03
- (ii) $P_n(-1)^n = (-1)^n$. 04
- (b)** (i) Solve the differential equation $y'' + xy = 0$ by the power series method. 03
- (ii) State Rodrigue's Formula and using it compute $P_0(x), P_1(x)$. 04
- OR**
- Q.4 (a)** (i) Solve the differential equation $xdy - ydx = \sqrt{x^2 + y^2}$. 03
- (ii) Solve : $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$. 04
- (b)** (i) If $y_1 = x$ is one solution of $x^2 y'' + xy' - y = 0$ then find the second solution. 03
- (ii) Solve : $(2x+5)^2 \frac{d^2 y}{dx^2} - 6(2x+5) \frac{dy}{dx} + 8y = 6x$. 04
- Q.5 (a)** (i) Find half range cosine series for the function $f(x) = e^x$ in interval $[0, 2]$. 03
- (ii) Express the function $f(x) = x - x^2$ as a Fourier series in interval $[-\pi, \pi]$. 04
- (b)** (i) Evaluate: $\int_0^1 (x \log x)^3 dx$. 03
- (ii) By using the relation between Beta and Gamma function prove that 04
- $$\beta(m, n) \beta(m+n, p) \beta(m+n+p, q) = \frac{\overline{m} \overline{n} \overline{p} \overline{q}}{\overline{m+n+p+q}}.$$
- OR**
- Q.5 (a)** Solve completely the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ representing the vibrations of a string of length l fixed at both ends given that, 07
- $$y(0, t) = y(l, t) = 0, y(x, 0) = f(x), \frac{\partial y}{\partial t}(x, 0) = 0, 0 < x < l.$$
- (b)** Find the Fourier Transform of the function f defined as follows: 07
- $$f(x) = \begin{cases} x & |x| < a; \\ 0 & |x| > a. \end{cases}$$
