

GUJARAT TECHNOLOGICAL UNIVERSITY
BE – SEMESTER-VIII- EXAMINATION – SUMMER • 2014

Subject Code: 181604**Date: 27-05-2014****Subject Name: Design and Analysis of Algorithm****Time: 10.30 am to 01.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) What is an algorithm? What do you mean by correct algorithm? What do you mean by instance of a problem? List out the criteria that all algorithms must satisfy. On what bases will you consider algorithm A is better than algorithm B? **06**

(b) What do you mean by Polynomial time complexity and Logarithmic time complexity? Which one is higher? What does $O(1)$ mean? **04**

(c) Arrange following functions n in increasing order: **04**

$$2^n, \log_2 n, n^3, n^{\log_2 n}, 2^{\log_2 n}, n^2 \log_2 n, e^{\log_2 n}, 3^n, 2^{2^n}, \frac{1}{n}, n \log_2 n$$

Q.2 (a) Define Time Complexity and Space Complexity. Why we are generally concerned with Time Complexities than Space Complexities? What is a major contributor for inefficiency of a loop? What will be theta notation for: $4n^3+5n+6$? **07**

OR

(a) Explain why the statement, “The running time of algorithm A is at least $O(n^2)$ ” is meaningless. Also explain what is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine? **07**

(b) What is Time Complexity? What is significance of Time Complexity in Algorithm Analysis? Provide methods and ways to measure the time complexity? What is significance of Time Complexity in algorithm analysis? Explain Limit rules for O , Ω , Θ notations. Express the function $n^3/1000-100n^2-100n+3$ in terms of Θ notation and prove that. **07**

OR

(b) Write down the algorithm to find minimum of N numbers and find its complexity. Also derive the worst case complexity of searching a Key from a binary search tree (balanced) having n nodes. **07**

Q.3 (a) Write an algorithm to find the maximum and minimum element in an array A storing n integers. What is the running time of this algorithm for computing the maximum element in an array of integers? **06**

(b) What do you mean by “eventually non decreasing function”? Give an example for the same. **04**

(c) What is optimal binary search tree problem? Explain in brief. **04**

OR

Q.3 (a) What is Space Complexity? How algorithms can be analyzed in terms of space complexity? Will it depend on type of Instance (input) or will change? **04**

(b) Solve the following recurrence equations. **06**

1. $T(n) = 7T(n/3) + n^2$

2. $T(n) = 4T(n/2) + \log n$

- (c) Differentiate decision problem and optimization problem. Is $P = NP$? List out the problems which can solve in polynomial time. List out the problems which cannot solve in polynomial time. **04**

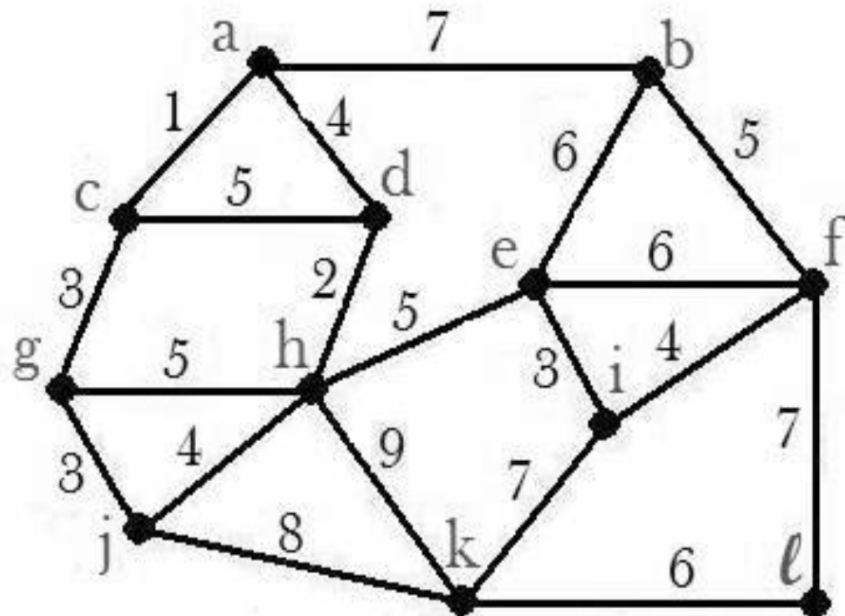
- Q.4** (a) Is $3\log n + \log \log n$ is $O(\log_{10} n)$? Is $3\log n + \log \log n$ is $\Omega(\log_{10} n)$? **04**
 (b) Differentiate between Greedy, Dynamic Programming and Divide and Conquer method. **06**
 (c) Write an algorithm for depth first search of a graph and explain with an example. Write an algorithm for breadth first search of a graph and explain with an example. **04**

OR

- Q.4** (a) Given two sequences of characters: $P = \langle \text{MLNOM} \rangle$ & $Q = \langle \text{MNOM} \rangle$. Obtain the longest common subsequence. **07**

- (b) Define String-matching problem. Compare all string matching algorithms. **07**

- Q.5** (a) There is a network given in the figure below as a highway map and the number recorded next to each arc as the maximum elevation encountered in traversing the arc. A traveler plans to drive from node 1 to 12 on this highway. The traveler dislikes high altitudes and so would like to find a path connecting node 1 to 12 that minimizes the maximum altitude. Find the best path for the traveler using a MST.
 (Graph do be drawn for Kruskal or Prims algorithm for minimum spanning tree) **07**



- (b) Check equalities(True/False): **07**

$$5n^2 - 6n \in \Theta(n^2)$$

$$n! \in O(n^n)$$

$$2n^2 2^n + n \log n \in \Theta(n^2 2^n)$$

$$\sum_{i=0}^n i^2 \in \Theta(n^3)$$

$$n^2 \in \Theta(n^3)$$

$$2^n \in \Theta(2^{n+1})$$

$$n! \in \Theta((n+1)!)$$

OR

- Q.5** (a) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For input of size n , insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \lg n$ steps. For which values of n does insertion sort beat merge sort? **05**

- (b) Given a knapsack having maximum weight capacity $W=4$, and number of items available are three, such that **05**
 $S=3$
 $w_i = \langle 1, 3, 4 \rangle$
 $v_i = \langle 3, 4, 5 \rangle$
Fill the knapsack using dynamic programming such that knapsack should not exceed its maximum capacity and it should have maximum profit value. Is dynamic programming a Top-Down or a Bottom-Up technique? Why?
Write the recurrence for solving Tower of Hanoi problem having n rings and 3 rods and **04**
- (c) solve the recurrence.
