Seat No.: _____ **Enrolment No._____**

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (OLD) - EXAMINATION – SUMMER 2017

Subject Code: 140001 **Date: 30/05/2017**

Subject Name: Mathematics-4

Time: 10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)** (i)	*Find the fifth root of unity*	**04**
	(ii)	*Prove that* $\sinh^{-1}(\tan\theta) = \log\{\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\}$	**03**
	(b)(i)	*Define Harmonic function. Show that the function* $u(x,y) = ax^2 - y^2 + xy$ *is harmonic.Determine its harmonic conjugate v(x,y) and the analytic function f(z)=u+iv*	**04**
	(ii)	*show that the function* $f(z) = \sqrt{\lvert xy\rvert}$ *is not analytic at theorigin but C-R equation are satisfied.*	**03**
Q.2	**(a)** *(i)*	*find the radii of convergent of the power series* $\sum_{n=0}^{\infty}(n+2i)^n z^n$.	**04**
	(ii)	*Define mobius transformation. Also find the fixed points and normal form of the following bilinear transformation w* $= \frac{z}{z-2}$	**03**
	(b) *(i)*	*Find and sketch the image of the region* $x \geq 1$ *under the transformation w* $= \frac{1}{z}$	**04**
	(ii)	*State the necessary condition for f(z) to be analytic. Determine f(z) = u+ iv whose imaginary part is* $log(x^2+y^2)+x-2y$	**03**
		OR	
	(b)(i)	ind the image of the region $\lvert z-3i\rvert = 3$ under the mapping $W=\frac{1}{z}$	**04**

	(ii)	*Expand f(z) =* $\frac{1}{z(z^2-3z+2)}$ *about z=0 for the reigons*		**03**
		(i) $0 < \|z\| < 1$ (ii) $1 < \|z\| < 2$		
Q.3	**(a) (i)**	*State and prove cauchy's Integral formula.*		**04**
	(ii)	Define pole, Residue ,Essential singularity.		**03**
	(b) (i)	*Prove that* $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$		**04**
	(ii)	*Obtain the Laurent series for the domain* $\|z-1\| > 1 \quad for \quad f(z) = \frac{z-1}{z^2}$		**03**
		OR		
Q.3	**(a) (i)**	Evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}$ dz where C is the circle $\|z\|$ =		**04**
	(ii)	3		**03**
		Find the nature and location of singularities of the function $\frac{1}{z^2-z^6}$		
	(b) i	*Evalute* $\oint_c \frac{z}{z^2+1} dz$ *, where c is the circle* $\|z+\frac{1}{2}\| = 2$		**04**
	(ii)	*Determine the residue of the given function at each pole* $\frac{1-e^z}{z^3}$		**03**
Q.4	**(a)(i)**	*Find the cubic polynomial for y(0)=1,y(1)=0, y(2)=1 and y(3)=10*		**04**
	(ii)	*Hence find y(4).*		**03**
		Solve the following system of equation by Gauss-Sidel method		
		$10x_1 + x_2 + x_3 = 6$		
		$x_1 + 10x_2 + x_3 = 6$		
		$x_1 + x_2 + 10x_3 = 6$		
	(b) (i)	*State simpsom's* $\frac{3}{8}$ *Rule and evaluate* $\int_0^1 \frac{dx}{1+x^2}$ *taking h=* $\frac{1}{6}$		**04**

Find a root of the equation $x^3 - x - 11 = 0$

(ii) *Correct to four decimal places using bisection method.* **03**

OR

Q.4 **(a)(i)** Consider following tabular values. **04**

X	*25.0*	*25.1*	*25.2*	*25.3*	*25.4*	*25.5*	*25.6*
y	*3.205*	*3.217*	*3.232*	*3.245*	*3.256*	*3.268*	*3.280*

Determine the area bounded by the given curve and X-axis between x=25 to x=25.6 by Trapezodial rule.

(ii) Evaluate $\int_1^3 \sin x \, dx$ using gauss quadrature of five points. Compare the results with analytic value. **03**

(b)(i) *Evaluate the integral* $\int_{-2}^{16}(1+x^2)^{\frac{3}{2}}\, dx$ *by Simpson's* **04**

$\frac{1}{3}$*Rule with taking 6 subintervals.*

(ii) **03**

Solve the following linear system of equations by Gauss-elimination method

$$x + 4y - z = -5$$

$$x + y - 6z = -12$$

$$3x - y - z = 4$$

Q.5 **(a)(i)** *Using Newton – Raphson method find a root of the equation* $x^3 - 5x + 3 = 0$ **04**

Correct to three decimal places.

(ii) *Determine the interpolating polynomial of degree three using Langranges interpolation formula.* **03**

X	*0*	*1*	*3*	*4*
Y	*-12*	*0*	*12*	*24*

(b)(i) *Use Jacobi's method to solve the equation* **04**

4x+y+3z=17 ,x+5y+ z =14, 2x-y+8z=12.

(ii) *Use Runge-kutta second order method of solve intial value problem* $\frac{dy}{dx} = -y\,, y(0) = 1\; for \quad x_1 = 0.2$ *and* $x_2 = 0.$ **03**

OR

Q.5 **(a)(i)** Usefourth order Runge- Kutta method to find y(0.2) with h=0.1, given that **04**

$$10\frac{dy}{dx} = x^2 + y^2\,, y(0) = 1$$

ii *power method to find the largest eigen value and corresponding eigen vector of the matrix* **03**

$$A=\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

(b)(i) *Find the real root of* $\sin x = 1 + x^3$, *correct three decimal places, using Newton-Raphson method.* **04**

(ii) *Use Regula-Falsi method to find a real root of the equation* $\log x - \cos x = 0$ *correct to three decimal places.* **03**

****