Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. VIIth Semester–Examination – Nov- 2011

Subject code: 172503

Subject Name: Optimization Methods

Date:24/11/2011 **Time: 10:30 am – 01:00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Paper rolls are produced by a paper mill in standard rolls of width 16 feet. (By width we mean the dimension perpendicular to the circular base when the roll is in its unrolled form). The mill receives orders from its customers for 200 rolls of width 3 feet, 50 rolls of width 4 feet, and 140 rolls of width 5 feet. It is assumed that all four types of roll have the same unrolled length. **07**

To satisfy the orders, the mill will have to cut a certain numbers of standard rolls in to smaller rolls. Various efficient cutting patterns are possible. E.g. a standard roll can be cut in to three rolls of width 3 feet and one smaller roll of width 5 feet, resulting in a trim loss of 2 feet. Another cutting pattern consists of two smaller rolls of width 3 feet, one smaller roll of width 4 feet and one smaller roll of width 5 feet, the trim los here is 1 feet.

Determine the entire efficient cutting patterns. Then set up the LPP to determine the minimum number of standard rolls to be cut so as to satisfy the customer orders.

(b) Solve the following LPP using dual simplex method **07**

Min $Z = -6x_1 - 7x_2 - 3x_3 - 5x_4$

Subject to

$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$

x_1, x_2, x_3, and x_4 are all non − negative

Q.2 **(a)** A company dealing in four different products is having a following LPP to solve. The optimal solution for the problem is given in table below. The variable x_1, x_2 etc. shows the production of four different products. **07**

Maximize $Z = 45x_1 + 100x_2 + 30x_3 + 50x_4$

Subject to

$$7x_1 + 10x_2 + 4x_3 + 9x_4 \leq 1200$$
$$3x_1 + 40x_2 + x_3 + x_4 \leq 800$$

x_1, x_2, x_3, and x_4 are all non − negative.

Optimal Solution Table.

	C_j	45	100	30	50	0	0	
C_B	Basis	x_1	x_2	x_3	x_4	s_1	s_2	b
30	x_3	5/3	0	1	7/3	4/15	-1/15	800/3
100	x_2	1/30	1	0	-1/30	-1/150	2/75	40/3
	$C_j - Z_j$	-25/3	0	0	-50/3	-22/3	-2/3	

The R&D department of the company has proposed a new product which can give a profit of Rs. 170 per unit and consume 10 units from each of the existing

resource. What can you say about the proposal of the R&D department? Support your argument with data.

(b) Consider two different types of foodstuffs F_1 and F_2. Assume that these foodstuffs contain vitamins V_1, V_2, and V_3. Minimum daily requirements of these vitamins are 1 mg of V_1, 50 mg of V_2 and 10 mg of V_3. Suppose the foodstuff F_1 contains 1 mg of V_1, 100 mg of V_2 and 10 mg of V_3 whereas foodstuff F_2 contains 1 mg of V_1, 10 mg of V_2 and 100 mg of V_3 and cost of one unit of foodstuff F_1 is Rs. 2.00 and that of F_2 is Rs. 2.50, find the minimum cost diet that would supply the body at least the minimum requirement of each vitamin. You are free to use any method for solving the above LPP. **07**

OR

(b) Solve the following LPP using Two phase method **07**

Maximize $Z = 3x_1 + x_2$

Subject to

$$3x_1 + 2x_2 \leq 24$$
$$4x_1 - x_2 \geq 8$$
$$x_1 - 2x_2 = 0$$

x_1 and x_2 are non − negative

Q.3 **(a)** In the problem of assignment of five operators (I to V) to five machines (A to E), the assignments costs are given below. **07**

	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	3	6
C	10	7	2	2	2
D	5	11	9	7	12
E	7	9	10	4	12

Assign the operator to different machines so that total cost is minimized.

(b) The unit cost of transportation from site *i* to site *j* are given below. At site *i* = 1, 2, 3, stocks of 150, 200 and 170 units respectively are available. 300 units are to be sent to site 4 and the rest to site 5. Find the cheapest way of doing this. **07**

	1	2	3	4	5
1	-	3	4	10	7
2	1	-	2	16	6
3	7	4	-	12	13
4	8	3	9	-	5
5	2	1	7	5	-

OR

Q.3 **(a)** Western Gujarat Company has taken the third floor of a multi-storied building for rent with a view to locate one of their zonal offices. These are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows; some are closer to the washroom or to the canteen or secretarial pool. The rooms are all of different sizes and shapes. Each of the five managers was asked to rank their room preferences among the rooms 301, 302, 303, 304 and 305. The recorded preferences are given below. **07**

Managers				
M_1	M_2	M_3	M_4	M_4
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
	301	305	303	
		302		

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they left off these from the list. Assuming that their preferences can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is minimum.

(b) The transportation problem for which the cost, origin availabilities and destination requirements are given below. Obtain the solution to minimize total transportation cost. **07**

	D_1	D_2	D_3	D_4	D_5	D_6	Availability
O_1	1	2	1	4	5	2	30
O_1	3	3	2	1	4	3	50
O_1	4	2	5	9	6	2	75
O_1	3	1	7	3	4	6	20
Requirements	20	40	30	10	50	25	

Q.4 **(a)** Two persons X and Y work on a two-station assembly line. The distribution of activity times at their stations are as follows. **07**
Simulate operation for eight items assuming a sequence of operation as X to Y and Y must wait until X completes is task for first item. What is the average waiting time of items for Y. use the following random numbers
For X: 83, 70, 06, 12, 59, 46, 54 and 04.
For Y: 51, 99, 84, 81, 15, 36, 12 and 54.
Also determine the numbers of items waiting between two stations and average production rate.

Time (sec.)	Time freq. for X	Time freq. for Y
10	4	2
20	7	3
30	10	6
40	15	8
50	35	12
60	18	9
70	8	7
80	3	3

(b) All the trucks travelling on Expressway between Ahmedabad and Vadodara are required to stop at a weigh station. Trucks arrive at the weigh station at a rate of 120 per eight hour day (Poisson Distributed), and the station can weigh, on the average, 140 trucks per day (Exponentially Distributed). **07**

(i) Determine the average number of trucks waiting, the average time spent at the weigh station by each truck, and the average waiting time before being weighed for each truck.

If the truck drivers find out they must remain at the weigh station longer than 15 minutes on the average, they will start taking a different route or travelling at night, thus depriving the state of taxes. The state of Gujarat estimates it loses Rs. 10,000 in taxes per year for each extra minute (over 15 minute) that trucks must remain at the weigh station. A new set of scales would have the same service capacity as the present set of scales and it is assumed that arriving trucks would line up equally behind the two sets of scales. It would cost Rs. 50,000 per year to operate the new scales. Should the state install the new set of scales?

OR

Q.4 (a) A dentist schedules all his patients for 15 minutes appointments. Some of the patients take more or less than 15 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needed o complete the work. **07**

Category	Time Required	Probability
Filling	22	0.30
Crown	30	0.20
Cleaning	8	0.15
Extraction	22	0.15
Checkup	8	0.20

Simulate the dentist's clinic for 3 hours and average waiting time for the patients as well as the idleness of the doctor. Assume that all the patient show up at the clinic at exactly their schedule arrival times, starting at 8 a.m. The random numbers are: 40, 82, 11, 34, 25, 66, 17, 79, 50, 37, 89, and 06.

Q.4 (b) A typist at an office of a company receives on the average 20 letters per day for typing. The typist work 8 hours a day and it takes on the average 20 minutes to type a letter. The cost of the letter waiting to be mailed is Rs. 0.80 per hour and the cost of the equipment plus salary of the typist is Rs. 45 per day. What is the average numbers of letters to be typed and to be mailed? **07**

In order to improve the typing service, the company has the choice to take lease of one of the two models of an automated typewriter. The daily cost and the resulting increase in efficiency of the typist are given below. What action should the company take to minimize the total daily cost of waiting letters to be mailed?

Model	Additional cost/day	Increase in typist's efficiency
I	Rs. 20	50%
II	Rs. 25	75%

Q.5 (a) In a game of matching coin with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails, and loses ½ unit of value when there are one head and one tail. Determine the payoff matrix, the bet strategy for each player and the value of the game to A **07**

(b) Solve the following game by using method of matrices **07**

		B		
		I	II	III
A	I	7	1	7
	II	9	-1	1
	III	5	7	6

OR

Q.5 (a) Reduce the following game to 2x2 game by using dominance and average dominance property and then solve the game. **07**

		B			
		B_1	B_2	B_3	B_4
	A_1	1	2	-1	2
A	A_2	3	1	2	3
	A_3	-1	3	2	1
	A_4	-2	2	0	-3

(b) In a well known children's game, each player says 'stone' or 'scissors' or 'paper'. If one says 'stone' and the other 'scissors', then the former wins a rupee. **07**

Similarly ‘scissors’ beats ‘paper’ and ‘paper’ beats ‘stones’, i.e., the player calling the former word wins a rupee. If the two players name the same item, then thee is a tie i.e., there is no payoff. Write down the payoff matrix, find the value of the game and hence write down the optimal strategies for both the players.

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