

**GUJARAT TECHNOLOGICAL UNIVERSITY****B. E. - SEMESTER – III • EXAMINATION – WINTER 2012****Subject code: 130001****Date: 09-01-2013****Subject Name: Mathematics - III****Time: 10.30 am – 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a)** Find a Fourier Series for  $f(x) = x^2$ , where  $0 \leq x \leq 2\pi$  **05****(b)** If  $f(x) = \pi + x$ ,  $-\pi < x < 0$  **05**

$$= \pi - x, \quad 0 < x < \pi$$

and  $f(x) = f(x + 2\pi)$ , for all  $x$  then expand  $f(x)$  in a Fourier Series.**(c)** Find a half range sine series for  $f(x) = \pi - x$ ,  $0 < x < \pi$  **04****OR****(c)** Find a Fourier Series for a periodic function  $f(x)$  with a period 2, where **04**

$$f(x) = -1, \quad -1 < x < 0$$

$$= 1, \quad 0 < x < 1$$

**Q.2 (a)** Find the inverse Laplace transforms of **05**

$$(i) \frac{5s^2 + 3s - 16}{(s-1)(s+3)(s-2)} \quad (ii) \frac{s^3}{s^4 - 81}$$

**(b)** State the Convolution theorem on Laplace transforms and using it find **05**

$$L^{-1} \left[ \frac{1}{s(s^2 + 4)} \right]$$

**(c)** Find the Laplace transforms of (i)  $\cos^2 2t$  and (ii)  $t^3 \cosh 2t$  **04****OR****(c)** Find the Laplace transforms of the half wave rectifier **04**

$$f(t) = \sin \omega t, \quad 0 < t < \frac{\pi}{\omega} \quad \text{and} \quad f(t) = f\left(t + \frac{2\pi}{\omega}\right)$$

$$= 0, \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$$

**Q.3 (a)** Define Beta function. **05**

$$\text{Prove that (i) } \int_0^1 \frac{1}{2} = \sqrt{\pi} \quad (ii) B(m, n) = B(m, n+1) + B(m+1, n)$$

**(b)** State the Duplication formula. **05**

$$\text{Show that } \int_0^\infty \sqrt{x} e^{-x^2} dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$$

**(c)** Show that (i)  $\int_{-a}^a (a+x)^{m-1} (a-x)^{n-1} dx = (2a)^{m+n-1} B(m, n)$  **04**

$$(ii) \int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5} B\left(\frac{2}{5}, \frac{1}{2}\right)$$

OR

- Q.3** (a) State the necessary and sufficient conditions to be exact differential equation. Using it, solve  $x^2y dx - (x^3 + y^3)dy = 0$  05
- (b) Using the method of variation of parameters, solve  $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$  05
- (c) Solve (i)  $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$  04
- (ii)  $\frac{dy}{dx} + y \tan x = \sin 2x$
- Q.4** (a) Using the method of undetermined coefficients, solve 05
- $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$
- (b) Solve  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left( x + \frac{1}{x} \right)$  05
- (c) Solve  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = \frac{e^x}{x^2}$  04

OR

- Q.4** (a) (i) If one of the solutions of  $x^2y'' - 4xy' + 6y = 0$  is  $y_1 = x^2, x > 0$  then determine its second solution. 05
- (ii) Solve :  $y''' - y'' + 100y' - 100y = 0$
- (b) Prove that, (i)  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$  (ii)  $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{\sin x}{x} - \cos x \right)$  05
- (c) Show that (i)  $J_{n-1}(x) - J'_n(x) = \frac{n}{x} J_n(x)$  (ii)  $J_0(0) = 1$  04
- Q.5** (a) Express  $f(x) = x^4 + 2x^3 + 2x^2 - x - 3$  in terms of Legendre's polynomials. 05
- (b) Find a power series solution of  $\frac{d^2y}{dx^2} + y = 0$  05
- (c) Classify the singularities for following differential equations 04
- (i)  $2x^2 \frac{d^2y}{dx^2} + 6x \frac{dy}{dx} + (x+3)y = 0$
- (ii)  $x(x+1)^2 y'' + (2x-1)y' + x^2y = 0$

OR

- Q.5** (a) Solve two dimensional Laplace's equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ , using the method separation of variables. 05
- (b) A rod of length L with insulated side is initially at uniform temperature  $100^\circ\text{C}$ . Its ends are suddenly cooled at  $0^\circ\text{C}$  and kept at that temperature. Find the temperature  $u(x, t)$ . 05
- (c) Find the Fourier transform of  $f(x) = xe^{-x}, x > 0$  04
- $= 0, x < 0$

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