Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – IV • EXAMINATION – WINTER 2012

Subject code: 140001 **Date: 30/12/2012**

Subject Name: Mathematics-IV

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Use the ϵ-δ definition of limit to show that $\lim_{z\to 3i}(3x + iy^2) = 9i$, where $z = x + iy$. **03**

(b) Prove that $|\exp(-2z)| < 1$ if and only if $Re\ z > 0$. **01**

(c) Show that $\cos(i\bar{z}) = \overline{\cos(iz)}$ for all z. **02**

(d) Expand $\cosh(z_1 + z_2)$. **02**

(e) Find all roots of the equation $\log z = i\pi/2$. **02**

(f) Given the data below, find the isothermal work done on the gas as it is compressed from V_1 =22 L to V_2 =2 L. Use $W = -\int_{V_1}^{V_2} PdV$. **04**

V, L	2	7	12	17	22
P, atm	12.20	3.49	2.04	1.44	1.11

Use Trapezoidal rule.

Q.2 **(a)** **(i)** Find the analytic function $f(z) = u + iv$ if **03**

$$u - v = (x - y)(x^2 + 4xy + y^2).$$

(ii) Choosing $x_0 = [1, 1, 1]^T$, writing $x_{i+1} = Ax_i$, and assigning $x_3 = x, x_4 = y$, apply the power method to find eigen value of matrix A, compute the Rayleigh quotient and an error bound at this stage, where **04**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

(b) **(i)** Find the bilinear transformation which maps the points $1, -1, \infty$ onto the points $1 + i, 1 - i, 1$ respectively. Also find its fixed points. **03**

(ii) Find a zero of the function $f(x) = x^3 - \cos x$, with starting point $x_0 = +1$, by Newton-Raphson Method. Could $x_0 = 0$ be used for this problem? **04**

OR

(b) **(i)** Show that when $Im\ z_0 < 0$, the transformation $w = e^{i\pi}\frac{z - z_0}{z - \bar{z}_0}$ maps the lower half plane $Im\ z \leq 0$ onto the unit disk $|w| \leq 1$. **03**

(ii) Use bisection method to find a root of the equation $x^3 + 4x^2 - 10 = 0$ in the interval [1, 2]. Find the relative percentage error at each iteration. Use four iterations. **04**

Q.3 **(a)** **(i)** State and prove Cauchy-Riemann conditions for a function $f(z) = u(x, y) + iv(x, y)$ to be analytic. **03**

(ii) Let a function $f(z)$ be analytic in a domain *D*. Prove that $f(z)$ must be constant in D in each of the following cases: (a) if $f(z)$ is real valued for all *z* in *D*, (b) if $\overline{f(z)}$ is analytic in *D*. **04**

(b) **(i)** Use residues to evaluate the integrals of the function $\frac{\exp(-z)}{z^2}$ around the circle $|z| = 3$ in the positive sense. **03**

(ii) The shear stress in kips, per square foot (ksf) for 5 specimens in a clay stratum are: **04**

Depth, m	1.9	3.1	4.2	5.1	5.8
Stress, ksf	0.3	0.6	0.4	0.9	0.7

Use Newton's divided-difference-interpolating polynomial to compute the stress at 4.5 m depth.

OR

Q.3 **(a)** **(i)** The function $f(z) = \begin{cases} \bar{z}^2/z & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$ satisfies the Cauchy-Riemann equations at the origin, but $f'(0)$ fails to exist. **03**

(ii) Derive the Taylor's series representation $\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}$, where $(|z-i| < \sqrt{2})$. **04**

(b) **(i)** Show that the singular point of the function $f(z) = (1 - \cosh z)/z^3$ is a pole. Determine the order m of that pole and the corresponding residue. **03**

(ii) Compute $f(4)$ from the tabular value given **04**

x	2	3	5	7
$f(x)$	0.1506	0.3001	0.4517	0.6259

using Lagrange interpolating polynomial.

Q.4 **(a)** Evaluate the integral $\int_{-2}^{6}(1+x^2)^{3/2}dx$ by the Gaussian formula for $n = 3$. **03**

(b) Given that $y = 1.3$ when $x = 1$ and $\frac{dy}{dx} = 3x + y$. Use second order Runge-Kutta method(i.e. Heun method) to approximate y when $x = 1.2$. Use step size 0.1. **03**

(c) Apply Gauss elimination method to solve the following system of simultaneous linear equations: **04**

$$2x + y - z = 1$$
$$5x + 2y + 2z = -4$$
$$3x + y + z = 5$$

(d) Describe geometrically the transformation $w = \frac{1}{z}$. State why it transforms circles and lines into circles and lines. **04**

OR

Q.4 **(a)** Evaluate the integral $\int_{-2}^{6}(1+x^2)^{3/2}dx$ by Simpson's 1/3rd rule with taking 6 subintervals. Use four digits after decimal point for calculations. **03**

(b) Show that the transformation $w = \sin z$ maps the top half $(y > 0)$ of the line $x = c_1$ $\left(-\frac{\pi}{2} < c_1 < 0\right)$ in a one to one manner onto the half $(v > 0)$ of the left hand branch of hyperbola $\frac{u^2}{\sin^2 c_1} - \frac{v^2}{\cos^2 c_1} = 1$. **03**

(c) Check whether the following system is diagonally dominant or not. If not, rearrange the equations so that it becomes diagonally dominant and hence solve the system of simultaneous linear equations by Gauss-Seidel method. **04**

$$-100y + 130z = 230$$
$$-40x + 150y - 100z = 0$$
$$60x - 40y = 200$$

	(d)	Solve the differential equation $\frac{dy}{dx} = x + y$, with fourth order Runge-Kutta method, where $y(0) = 1, x = 0$ to $x = 0.2$ to with $h = 0.1$.	**04**
Q.5	**(a)**	Show that when $0 < \lvert z-1 \rvert < 2$, $$\frac{z}{(z-1)(z-3)} = \frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}}$$	**05**
	(b)	Evaluate $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta}$.	**05**
	(c)	Determine the angle of rotation at the point $z = 2 + i$ when the transformation is $w = z^2$, and illustrate it for some particular curve. Show that the scale factor of the transformation at that point is $2\sqrt{5}$.	**04**

OR

Q.5	**(a)**	What is the largest circle within which the Maclaurin series for the function $\tanh z$ converges to $\tanh z$? Write the first two nonzero terms of that series.	**05**
	(b)	Find the value of the integral $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ taken counterclockwise around the circle $c: \lvert z-2 \rvert = 2$.	**05**
	(c)	Use Rouche's theorem to determine the number of zeros of the polynomial $z^6 - 5z^4 + z^3 - 2z$ inside the circle $\lvert z \rvert = 1$.	**04**

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