Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – WINTER 2013

Subject Code: 110014 **Date: 23-12-2013**

Subject Name: Calculus

Time: 10:30 am – 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt the following questions.

(1) Expand $\sin(\frac{\pi}{4}+\theta)$ in power of θ . Hence find the value of Sin 46°. **03**

(2) Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$. **02**

(3) Check the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. **02**

(b) Attempt the following questions.

(1) Using reduction formula, evaluate $\int_0^{\pi}(x\sin^5 x\cos^4 x)dx$. **03**

(2) Find $\frac{dy}{dx}$ if $y=\int_1^{x^3}\sin t dt$. **02**

(3) Test the convergence of $\sum_{n=1}^{\infty}\frac{\ln n}{n^{3/2}}$. **02**

Q.2 **(a)** Attempt following questions.

(1) If $x^x y^y z^z = a$ Show that at $x=y=z, \frac{\partial^2 z}{\partial x \partial y}=-(x\log ex)^{-1}$. **04**

(2) Evaluate $\iint_S \sqrt{xy-y^2}\,dxdy$ Where S is a triangle with vertices (0, 0) (10, 1) (1, 1). **03**

(b) Attempt following questions.

(1) Using appropriate reduction formula to evaluate following. **04**

(i) $\int_0^{2a} x^{7/2}(2a-x)^{-1/2}dx$

(ii) $\int_0^{\pi}(1-\cos x)^2 dx$

(2) Expand $\log(1+e^x)$ in ascending power of x as far as term containing x^4. **03**

OR

(b) Attempt following questions.

(1) Find the points of inflection on the curve $y=(\log x)^3$. **04**

(2) Test for convergence the series whose n[th] term is $\left(1+\frac{1}{\sqrt{n}}\right)^{n^{(-3/2)}}$ **03**

Q.3 **(a)** Attempt following questions.

(1) If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}}\right)$ then prove that **04**

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = \frac{1}{144}\tan u(\tan^2 u - 11)$$

(2) If $u = f(r)$ where $x = r\cos\theta, y = r\sin\theta$ Prove that **03**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

(b) Attempt following questions.

(1) Find the shortest and longest distance from the point (1,2,-1) to the sphere $x^2 + y^2 + z^2 = 24$ **04**

(2) Expand e^{xy} at (1, 1) up to three terms, by taylor's theorem. **03**

OR

Q.3 **(a)** Attempt following questions.

(1) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ **04**

(2)If $f(x, y) = 0, \phi(y, z) = 0$ prove that $\frac{\partial f}{\partial y}\frac{\partial \phi}{\partial z}\cdot\frac{dz}{dx} = \frac{\partial f}{\partial x}\frac{\partial \phi}{\partial y}$ **03**

(b) Attempt following questions.

(1) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. **04**

(2) Find the equation of the tangent plane and normal line to the surface $\frac{x^2}{2} - \frac{y^2}{3} = z$ at the point (2, 3, -1). **03**

Q.4 **(a)** Attempt following questions.

(1) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}}\,dydx$ by changing the order of integration. **04**

(2) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dzdydx$. **03**

(b) Attempt following questions.

(1) Evaluate $\iint_R (x+y)^2\,dxdy$, where R is the parallelogram in the xy plane with vertices (1,0), (3,1), (2,2), (0,1) using the transformation $u = x + y$ and $v = x - 2y$. **04**

(2) Find the total area enclosed by lemniscate $r^2 = a^2\cos 2\theta$. **03**

Q.5 **(a)** Attempt following questions.

(1) Find the volume bounded by the xy plane, the paraboloid $2z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$. **04**

(2) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. **03**

(b) Attempt following questions.

(1) Show that the volume of spindle-shaped solid generated by the astroid **04**

$x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis is $\frac{32\pi a^3}{105}$.

(2) Change into polar co-ordinate and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\,dy\,dx$. **03**

Q.6 **(a)** Attempt following questions.

(1) Trace the curve $y^2(2a-x) = x^3$. **04**

(2) Discuss the convergence of integral $\int_{-2}^{2} \frac{dx}{x^2}$. **03**

(b) Attempt following questions.

(1) Discuss the continuity of $f(x,y) = \frac{x}{\sqrt{x^2+y^2}}, x \neq 0, y \neq 0$ and **04**

= 2, x = 0,y=0

at the origin.

(2) Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. **03**

Q.7 **(a)** Attempt following questions.

(1) Test the convergence $\sum_{n=1}^{\infty} ne^{-n^2}$ **04**

(2) Find $F'(0)$ for $F(x) = \int_3^{\sin t} \frac{1}{1+t^2}\,dt$ by Leibnitz's rule for integral. **03**

(b) Attempt following questions.

(1) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$. **04**

(2) Find the linearization of $f(x,y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at point (3,2) **03**

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