Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – WINTER 2013

Subject Code: 130001 **Date: 05-12-2013**

Subject Name: Mathematics-III

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed: **14**

(a) Show that $y = be^{x} + ce^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

(b) Solve the differential equation: $(1+x^2)dy = xydx$.

(c) Solve the initial value problem: $y'' + y' - 2y = 0,\ y(0) = 4$ and $y'(0) = -5$.

(d) Evaluate the integral $\int_0^\infty x^6 e^{-2x}dx$.

(e) Prove that $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$.

(f) Find the Laplace transform of $2t^3 + e^{-2t} + t^{\frac{4}{3}}$.

(g) Find the Inverse Laplace transform of $\frac{3(s^2-1)^2}{2s^5}$.

Q.2 **(a)** Solve the differential equation by the method of variation of perameter. $\frac{d^2y}{dx^2} + y = \sec x$ **07**

(b)(i) Solve the differential equation. $((D^3 + D^2 - D - 1)y = \cos 2x$ **04**

(ii) Find the orthogonal trajectories of the family $ay^2 = x^3$. **03**

OR

(b)(i) Solve the differential equation. $((D^2 + 4)y = x^2 + \sin 2x$ **04**

(ii) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$.Find the current i as a Function of time t given that I as the maximum current and i=0 when t=0. **03**

Q.3 **(a)** Given that $f(t) = \begin{cases} t+1, 0 \le t \le 2 \\ 3, t \ge 2 \end{cases}$ find $L\{f(t)\}$ and $L\{f'(t)\}$. **05**

(b) Find the Laplace transform of $\frac{1-e^t}{t}$ **05**

(c) Solve the differential equation $x\frac{dy}{dx} + y = x^3y^6$. **04**

OR

Q.3 **(a)** Using convolution theorem ,evaluate the following : $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right]$. **05**

	(b)	By using the method of Laplace transform solve the initial value problem: $y'' + 2y' + y = e^{-t}$, $y(0) = -1$ and $y'(0) = 1$.	**05**
	(c)	Solve the differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$.	**04**
Q.4	**(a)**	Obtain the Fourier series to represent the function $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$.	**05**
	(b)	Find the Fourier series for the function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$. Deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$	**05**
	(c)	Express the function $f(x) = \begin{cases} \sin x, 0 \le x \le \pi \\ 0, x > \pi \end{cases}$ as a Fourier sine integral and evaluate $\int_0^\infty \frac{\sin \lambda x \sin \pi\lambda}{1-\lambda^2} d\lambda$.	**04**

OR

Q.4	**(a)**	Develop f(x) in a Fourier series in the interval (0,2) if $f(x) = \begin{cases} x, 0 < x < 1 \\ 0, 1 < x < 2 \end{cases}$	**05**
	(b)	Find the half range cosine series for $f(x) = e^x$ in (0,1).	**05**
	(c)	Find the Fourier sine transform of $f(x) = e^{-2x} + e^{-3x}, x > 0$.	**04**
Q.5	**(a)**	Find a series solution of $y'' + y = 0$ near x = 0.	**05**
	(b)	Solve the following by the method of separation of variables: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$.	**05**
	(c)	Show that $p_n(-x) = (-1)^n p_n(x)$, hence find $p_n(-1)$	**04**

OR

Q.5	**(a)**	Find a series solution of differential equation $xy'' + 2y' + xy = 0$.	**05**
	(b)	A tightly stretched string with fixed ends x = 0 and $x = l$ is initially at rest in its equilibrium position i.e. $y(0,t) = 0 = y(l,t)$ for all t and y(x,0) = 0 For $0 \le x \le l$. If it set vibrating giving each point a velocity $3x(l-x)$ i.e $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3x(l-x)$ for $0 \le x \le l$, find the displacement y(x,t), where $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$.	**05**
	(c)	Show that $\int_{-1}^{1} \frac{p_n(x)}{\sqrt{1-2xt+t^2}} dx = \frac{2}{2n+1} t^n$.	**04**

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