

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III • EXAMINATION – WINTER 2013****Subject Code: 130002****Date: 05-12-2013****Subject Name: Advanced Engineering Mathematics****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a) (1) Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = \frac{1}{(x^2+1)^3}$ 03
 (2) Solve $(xy - 2y^2)dx - (x^2 - 3xy)dy = 0$ 04
- (b) Solve in series the equation $\frac{d^2y}{dx^2} + x^2y = 0$ 07
- Q.2 (a) Solve the equation by method of separation of variables $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, 07
 where $u(0, y) = 8e^{-3y}$
- (b) Solve $(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ in series 07
 OR
- (b) (1) Solve $\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$ 03
 (2) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$ 04
- Q.3 (a) Find the Fourier Series to represent e^x in the interval $(-\pi, \pi)$. 07
 (b) Express $f(x) = \frac{1}{2}(\pi - x)$ in a Fourier Series in the interval $0 < x < 2\pi$. 07
 OR
- Q.3 (a) Find the Fourier expansion for the function $f(x) = x - x^3$ in the interval $-1 < x < 1$. 07
 (b) Prove that in the range $(0, l)$ 07

$$x = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{n=1}^{\infty} \left(\frac{1}{(2n-1)^2} \right) \cos \frac{(2n-1)\pi x}{l}$$
 and deduce that

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots = \frac{\pi^4}{96}$$
- Q.4 (a) Prove that $L(t^n) = \frac{n!}{s^{n+1}}$ & $L(\cos at) = \frac{s}{(s^2 - a^2)}$ 07
 (b) (1) Find $L^{-1} \left\{ \frac{s}{s^4 + 4a^4} \right\}$ 03
 (2) find $L^{-1} \left\{ \frac{1}{s(s+a)^3} \right\}$ 04
 OR
- Q.4 (a) Solve the differential equation by Laplace Transform method 07
 $y'' + 4y' + 3y = e^{-t}, y(0) = y'(0) = 1$
- (b) If $L\{f(t)\} = \bar{f}(s)$, then show that $L\{tf(t)\} = -\frac{d}{ds}\{\bar{f}(s)\}$ use this 07
 result to obtain $L\{e^{at}t \sin at\}$

- Q.5 (a) (1) Define the following terms
- (i) Gamma function 02
 - (ii) Dirac Delta function 02

- (2) Form partial differential equation by eliminating the arbitrary function from
 $z = f(x^2 - y^2).$ 03

- (b) Find Fourier integral representation of the function 07

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$

OR

- Q.5 (a) (1) Form partial differential equation by eliminating the arbitrary function from 04

$$xyz = \phi(x + y + z).$$

- (2) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ 03

- (b) (1) Solve $pz - qz = z^2 + (x + y)^2$ 04

- (2) Solve $z = px + qy - 2\sqrt{pq}$ 03
