Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – WINTER • 2014

Subject Code: 130001 **Date: 06-01-2015**

Subject Name: Mathematics-III

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find Laplace transformation of following functions

(1) $\sin 2t \cos 2t$ **02**

(2) $\cos^3 2t$ **02**

(3) Unit step function **03**

(b) (1) Define order and degree of a differential equation. Find order and degree of differential equation $\sqrt{x^2 \frac{d^2y}{dx^2} + 2y} = \frac{d^3y}{dx^3}$. **04**

(2) Find out general solution of differential equation $e^x dx - e^y dy = 0$ **03**

Q.2 **(a)** Solve differential equations:

(1) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 18y = 0$ **03**

(2) $y'' + y' - 2y = 0$ with $y(0) = 4$ and $y'(0) = -5$ **04**

(b) (1) Solve $\frac{dy}{dx} + \tan x \cdot y = \cos x$ given that $y(0) = 2$ **04**

(2) Solve $\left(x^2y - 2xy^2\right)dx - \left(x^3 - 3x^2y\right)dy = 0$ **03**

OR

(b) (1) In usual notation, write series expression for $J_0(x)$ and $J_1(x)$ **03**

(2) Using series method, solve in sequence $y'' + x^2y = 0$ **04**

Q.3 **(a)** (1) Using convolution theorem, evaluate $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$ **04**

(2) Solve using Laplace transforms, $y''' + 2y'' - y' - 2y = 0$; where, $y(0) = 1$, $y'(0) = 2$, $y''(0) = 2$ **03**

(b) Obtain Fourier series for the function $f(x)$ given by **07**

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

OR

Q.3	**(a)**	(1) Solve differential equation using method of undetermined coefficients $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=3e^{-2x}$	**03**
		(2) Find general solution of $y''+9y=\sec 3x$ using method of variation of parameters.	**04**
	(b)	Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$	**07**
Q.4	**(a)**	(1) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$	**07**
	(b)	Find inverse Laplace transforms of following :	
		(1) $\frac{1}{s(s+1)}$	**03**
		(2) $\frac{5s+3}{(s-1)(s^2+2s+5)}$	**04**
		OR	
Q.4	**(a)**	Solve completely the equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$, representing vibration of string of length l, fixed at both ends, given that $y(0,t)=y(l,t)=0$; $y(x,0)=f(x)$ and $\frac{\partial y(x,0)}{\partial t}=0;\ 0<x<l$	**07**
	(b)	(1) Evaluate $\int_{-1}^{1}(1+x)^m(1-x)^n\,dx$, where $m,n>0$ are integers	**03**
		(2) Find Fourier Sine series of $f(x)=e^x$, $(0<x<L)$	**04**
Q.5	**(a)**	Obtain Series solution of $(1-x^2)y''-2xy'+2y=0$	**07**
	(b)	Find Fourier series of $f(x)=x^2, 0\le x\le 2\pi$. Deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.....=\frac{\pi^2}{12}$	**07**
		OR	
Q.5	**(a)**	(1) Find the ordinary points, regular singular points and irregular singular points of the differential equation $x^3y''+5xy'+3y=0$	**04**
		(2) In usual notations, prove $\int_{-1}^{1}P_m(x)P_n(x)\;dx=0,\ m\ne n$	**03**
	(b)	(1) Write down Rodrigue’s Formula and use it to find $P_0(x),\ P_1(x)$	**03**
		(2) In usual notations, prove that $\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$	**02**
		(3) Expand $\pi x-x^2$ in a half-range sine series in the interval $(0,\pi)$ up to first three terms.	**02**

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