

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III • EXAMINATION – WINTER • 2014****Subject Code: 130002****Date: 06-01-2015****Subject Name: Advanced Engineering Mathematics****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (1) Find the differential equation of the family of circles of radius r whose centre lies on the x -axis. **04**

(2) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ **03**

(b) Find the series solution of $y' - 2xy = 0$ **07**

Q.2 (a) Solve by method of separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$ **07**

(b) Solve in series the differential equation $4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0$ **07**

OR

(b) (1) Solve the Initial Value Problem $y'' - 9y = 0$; $y(0) = 2$, $y'(0) = -1$ **03**

(2) Solve $(D^2 + 16)y = x^4 + e^{3x} + \cos 3x$ **04**

Q.3 (a) (1) Find the Laplace transform of $f(t) = 0$; $0 \leq t \leq 3$ **03**
 $= 4$; $t \geq 3$

(2) Prove that $L(\sin h kt) = \frac{k}{s^2 - k^2}$ **04**

(b) (1) Find $L^{-1} \left\{ \frac{1}{s(s^2 + 4)} \right\}$ **03**

(2) Find $L^{-1} \left\{ \frac{3s^2 + 2}{(s+1)(s+2)(s+3)} \right\}$ **04**

OR

Q.3 (a) If $L\{f(t)\} = F(s)$ then show that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \{F(s)\}$; $n = 1, 2, 3, \dots$ **07**

and use this result find $L(t^2 \sin wt)$

(b) Solve the differential equation by Laplace Transform **07**

$$y'' + y = \sin 2t; \quad y(0) = 2, \quad y'(0) = 1$$

Q.4 (a) Find the Fourier series of $f(x) = x + |x|$; $-\pi < x < \pi$ **07**

(b) Find the Fourier expansion $f(x) = x^2 - 2$; $-2 \leq x \leq 2$ **07**

OR

Q.4 (a) Obtain the cosine series for the function $f(x) = e^x$ in the range $(0, l)$ **07**

(b) Find the Fourier series for the periodic function $f(x)$ **07**

$$f(x) = \begin{cases} -k; & \text{if } -\pi < x < 0 \\ k; & \text{if } 0 < x < \pi \end{cases} \quad f(x + 2\pi) = f(x)$$

$$\text{Hence deduce that } 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

Q.5 (a) (1) Define the following terms: **04**

(i) Beta function

(ii) Heaviside's function

(2) Eliminate the arbitrary function from the equation **03**

$$z = xy + f(x^2 + y^2)$$

(b) Using Fourier integral representation, show that **07**

$$\begin{aligned}\int_0^{\infty} \frac{\cos x\lambda + \lambda \sin x\lambda}{1+\lambda^2} d\lambda &= 0 && ; \quad x < 0 \\ &= \pi/2 && ; \quad x = 0 \\ &= \pi e^{-x} && ; \quad x > 0\end{aligned}$$

OR

- Q.5** **(a)** (1) Solve $(y+z)p + (z+x)q = x+y$ **04**
 (2) Solve $p^2 + q^2 = npq$ **03**
 (b) (1) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$ **04**
 (2) Solve $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$. **03**
