Seat No.: _________ **Enrolment No.______________**

GUJARAT TECHNOLOGICAL UNIVERSITY
BE SEM- I / II Winter Examination-Dec.-2011

Subject code: 110014 **Date: 23/12/2011**
Subject Name: Calculus
Time: 10.30 am -1.30 pm **Total marks: 70**

Instructions:
1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Do as directed **07**

(1) What curve is represented by the polar equation r = 1?

(2) Find $\lim_{(x,y)\to(5,-2)} (x^4 + 4x^3y - 5xy^2)$.

(3) For what values of r is the sequence $\{r^n\}$ convergent?

(4) Write reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$ where $n \in \mathbb{N}$ and $n \geq 2$.

(5) Is $\int_1^{\infty} \frac{1}{x^{\sqrt{2}}} dx$ convergent?

(6) Evaluate $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$.

(7) If f has a local minimum at (a,b), what can you say about its first order partial derivatives at (a,b)?

(b) Do as directed

(1) Sketch the polar curve $\theta = 1$. **02**

(2) Evaluate $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz\, dy\, dx$. **02**

(3) Find maxima and minima of the function **03**
$f(x, y) = x^3 + y^3 - 3x - 12y + 20$

Q.2 **(a)** (1) Write comparison test for improper integrals and using it show that $\int_0^{\infty} e^{-x^2} dx$ is convergent. **04**

(2) Find the volume of the solid obtained by rotating the region enclosed by the curves y = x and y = x^2 about the X-axis. **03**

(b) (1) Using Fundamental theorem of Calculus, find the derivative of the function $h(x) = \frac{d}{dx}\left(\int_1^{x^4} \sec t\, dt\right)$. Also evaluate $\int_0^1 x^4 (1 - x^2)^{3/2}\, dx$. **04**

(2) Using cylindrical shells, find the volume of the solid **03**

obtained by rotating about the X-axis the region under the curve $y=\sqrt{x}$ from 0 to 1.

Q.3 **(a)** (1) Determine whether the following series converge or diverge. Find the sum of the series if it converges. **04**

(i) $\sum_{n=1}^{\infty}[\tan^{-1}(n)-\tan^{-1}(n+1)]$

(ii) $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots$

(2) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$. **03**

(b) (1) Assuming the validity of expansion, prove that, for $0<x\le 2$, **04**

$$\log x=(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\cdots$$

(2) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$. **03**

Q.4 **(a)** (1) Find the Maclaurin series for the function $f(x)=\frac{1}{\sqrt{4-x}}$ and its radius of convergence. **04**

(2) Assuming the validity of expansion, show that $\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{\sqrt{2}}\left(1+\theta-\frac{\theta^2}{2!}-\frac{\theta^3}{3!}+\cdots\right)$. **03**

(b) (1) Show that the series $\frac{1}{1^p}-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\cdots$ converges for $p>0$. **04**

(2) Show that the sequence $\{a_n\}$ whose n^th^ term is $a_n=\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}$, $n\in$ □ , is monotonic increasing and bounded. Is it convergent? **03**

Q.5 **(a)** (1) If $u=f(x,y)$, where $x=e^s\cos t$ and $y=e^s\sin t$, show that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2\right]$. **04**

(2) Show that $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}} & ,\ (x,y)\ne(0,0)\\ 0 & ,\ (x,y)=(0,0)\end{cases}$ is continuous at the origin . **03**

(b) (1) Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point (3,1,-1). **04**

(2) If $u(x,y)=\log\left(\frac{x^2+y^2}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$ **03**

Q.6 **(a)** (1) Expand $e^x \tan^{-1} y$ about (1,1) upto the second degree in (x-1) and (y-1). **04**

(2) The base radius and height of a right circular cone are measured as 10 cm and 25 cm respectively, with a possible error in measurement of as much as 0.1 cm in each. Estimate the maximum error in the calculated volume of the cone. **03**

(b) (1) Find the equations of tangent plane and normal line at the point (-2,1,-3) to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$. **04**

(2) Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at point (2,1,0). **03**

Q.7 **(a)** (1) Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection and local maxima and minima. Use this information to sketch the curve. **04**

(2) Evaluate the integral $I = \int_0^1 \int_0^x \sqrt{x^2 + y^2}\; dydx$ by passing on to the polar coordinates. **03**

(b) (1) Evaluate $\iint (y - x)\, dxdy$ over the region E in the XY-plane bounded by the straight lines $y = x - 3,\ y = x + 1,\ 3y + x = 5,\ 3y + x = 7$. **04**

(2) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$. **03**

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