Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-IV Examination June- 2010

Subject code: 141701 **Subject Name: Control Theory**

Date: 19 / 06 /2010 **Time: 10.30 am – 01.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Please ask for one semi-log paper and one Nichols chart to supervisor, if you wish to attempt Q.5 (OR) in advance.**

Q.1 **(a)** Using the block diagram reduction techniques, find the closed loop transfer function of the system whose block diagram is given in **Fig. 1.** **08**

(b) Write short notes on open loop control systems and closed loop control systems. Discuss their advantages and disadvantages. **06**

Q.2 **(a)** Sketch the root loci of unity feedback control system on a graph paper using a suitable scale, whose open-loop transfer function is given below. Determine the range of gain for stability and the point at which it crosses the imaginary axis. Determine the value of gain K at the breakaway point. **07**

$$G(s) = \frac{K}{(S-1)(S^2 + 4S + 7)}$$

(b) Draw signal flow graph of the system shown in **Fig.1.** Obtain overall system transfer function using Mason's gain formula. **07**

OR

(b) (i) Find the number of roots in the right half of the s-plane using R-H criterion, for the characteristic equation given below: **07**

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

(ii) Consider a sixth-order system with the characteristic equation given below. Using R-H criterion, comment on its stability and find the roots on imaginary axis if any.

$$S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16s + 16 = 0$$

Q.3 **(a)** Write definitions of state and state variables. Derive expression of transfer function of the system which is represented in the following standard state space form: **06**

$$\dot{X} = AX + Bu \quad y = CX + Du$$

(b) Obtain the transfer function of the system defined by following state-space equations. **08**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

OR

Q.3 **(a)** Derive expression of response, c(t), of second order unity feedback system whose closed-loop transfer function is given below, for a unit step input as a function of time t and damping ratio ζ. Derive expression of c(t) for $\zeta=0$ and $\zeta=1$. **08**

$$T(s) = \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n + \omega_n^2}$$

(b) Derive expressions of (i) Rise time, t_r (ii) Peak time, t_p and (iii) Peak overshoot, M_p for the system of above Question 3(a). **06**

Q.4 **(a)** Consider the first order system with a controller as shown in **Fig. 2.** **07**

(i) Find the steady state error e_{ss} with unit step input using a proportional controller with gain K_p. **(see Fig. 2a)**

(ii) Find the steady state error e_{ss} with unit step input using an integral controller gain K_i. Comment on the results. **(see Fig. 2b)**

(b) Define thermal resistance and thermal capacitance. Explain thermal systems. **07**

OR

Q.4 **(a)** (i) A system has open-loop transfer function as given below. Find gain margin of this system without a sketch of Bode plots. **06**

$$G(S)H(S) = \frac{\omega_n^2}{S(S+2\zeta\omega_n)}$$

(ii) Explain frequency response and state its limitation.

(iii) Compare root locus technique and Nyquist stability criterion for stability study of control systems in short.

(b) Sketch the polar plot of the transfer function given below using suitable scale. Determine whether the plot crosses the real axis and imaginary axis. If so, determine the frequency at which these axes are crossed and corresponding magnitudes. **08**

$$G(s) = \frac{1}{(s+1)(1+0.2s)(1+0.4s)}$$

Q.5 **(a)** Comment on the stability of a system with an open-loop transfer function as given below, using Nyquist stability criterion. **07**

$$G(S)H(s) = \frac{K}{S(1+TS)}$$

(b) Explain constant-M circles and constant-N circles by deriving related expressions. Explain how resonant peak can be obtained. **07**

OR

Q.5 **(a)** Sketch Bode plots of a unity feedback control system having open-loop transfer function as given below. Determine gain margin and phase margin. **07**

$$G(S) = \frac{10}{S(1+0.1S)(1+0.05S)}$$

(b) Using the Bode plots obtained in the above problem; draw G(jω) on Nichols chart. Find bandwidth and resonant peak magnitude and the frequency at which it occurs. Sketch the closed-loop response on the same semi-log paper used for above problem. You may please use a different scale for magnitude if required. **07**

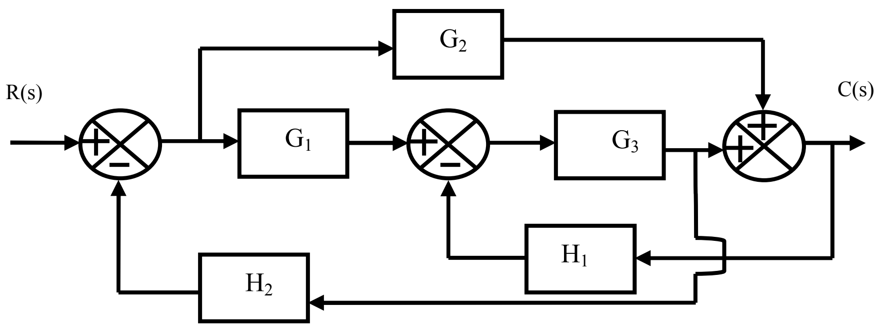

Figure 1 [Q.1 (a) and Q.2 (b)]

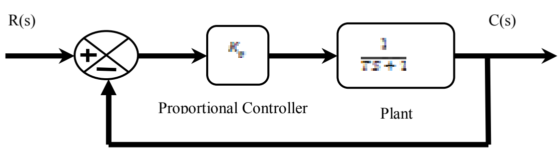

Figure 2a. [Q.4 (a) (i)]

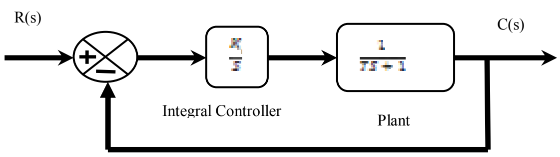

Figure 2b. [Q.4 (a) (ii)]