

GUJARAT TECHNOLOGICAL UNIVERSITYM. C. A. Semester - IST Examination –July- 2011

Subject code: 610003

Subject Name: Discrete Mathematics for Computer Science

Date:08/07/2011

Time: 02:30 pm – 05:00 pm

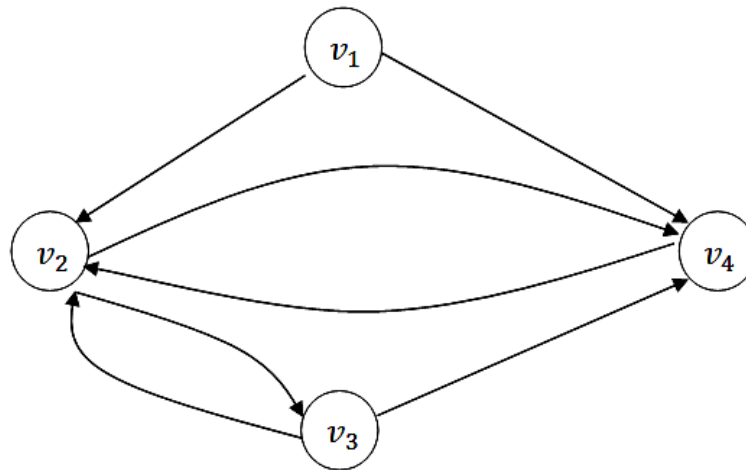
Total Marks: 70

Instructions:

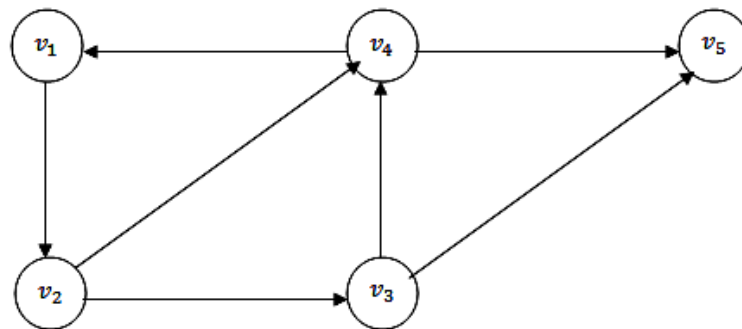
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) (i) Without constructing truth table prove that $\sim(p \vee q \vee r) \equiv \sim p \wedge \sim q \wedge \sim r$ **02**
 (ii) Give an example of a relation which is neither reflexive nor irreflexive. **02**
 (iii) Define group with example. Give an example of non-abelian group. **03**
- (b) (i) If predicate $R(x): x \text{ is a prime integer}$ then write a statement in English corresponding to the following statements:
 $R(10)$ and $\exists x R(x)$. **02**
 (ii) Define Boolean expression. **02**
 (iii) Define isomorphic graphs and give its example. **03**
- Q.2**
- (a) (i) Define statement formula. **02**
 (ii) Define Lattice as a partially ordered set with example. **02**
 (iii) Define sum-of-product canonical form. Write Boolean expression $x_1 * x_2$ in an equivalent sum-of-products canonical form in three variables x_1, x_2 and x_3 . **03**
- (b) Explain contradiction method and using it prove that $\sqrt{50}$ is an irrational number. **07**
- OR**
- (b) For an integer x prove that the following statements are equivalent: **07**
 $p: x \text{ is divisible by } 10.$
 $q: x \text{ is divisible by } 2 \text{ and } 5.$
 $r: x \text{ is an even number and } x \text{ is divisible by } 5.$
- Q.3**
- (a) Show that the operations of meet and join on a lattice are commutative, associative and idempotent. **07**
- (b) Use the Karnaugh map representation to find a minimal sum-of-products expression of $f(a, b, c, d) = \sum(0, 1, 2, 3, 13, 15)$. **07**
- OR**
- Q.3**
- (a) Define Distributive lattice. Give an example of non-distributive lattice. Also give an example to verify that every chain is a distributive lattice. **07**
- (b) Define Boolean function and Symmetric Boolean expression. Determine $a'bcd + a'c' + b'c'd' + ad'$ is symmetric or not. **07**
- Q.4**
- (a) State and prove DeMorgan's Law for Boolean Algebra. **07**
- (b) Find all the subgroups of S_4 generated by the permutations $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$. **07**
- OR**
- Q.4**
- (a) Describe the application of Boolean algebra to Relational Database with example. **07**

- Q.4 (b) Obtain the adjacency matrix A of the digraph given below. Also find the elementary paths of lengths 1 and 2 from v_1 to v_4 . 07



- Q.5 (a) Let $\langle G, * \rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n then prove that n is the least positive integer for which $a^n = e$. 07
- (b) Find the strong components of the following diagram. Also find its unilateral and weak components. 07



OR

- Q.5 (a) Define left coset and right coset. Find the left cosets of $\{[0], [3]\}$ in the group $\langle \mathbb{Z}_6, +_6 \rangle$. 07
- (b) Define a directed tree. Show by means of an example that a simple digraph in which exactly one node has indegree 0 and every other node has indegree 1 is not necessarily a directed tree. 07
