Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA. Sem- I^ST Regular / Remedial Examination January/ February 2011

Subject code: 610003

Subject Name: Discrete Mathematics for Computer Science

Date: 31 /01 /2011 **Time: 10.30 am – 01.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Define a Boolean algebra. Show that lattice $<P(A), \cup, \cap>$ is a Boolean algebra, where A = {a, b, c} and P(A) denotes its power set. What are the operations of meet and join in it? Draw the Hasse diagram of this Boolean algebra. **07**

(b) For the poset $<\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq>$ Draw the Hasse diagram and find : **07**

1) maximal elements and minimal elements
2) Greatest element and least element, if exists
3) Lower bounds of {1,3,4} and {2,3,4}
4) Upper bounds of {2,4} and {3,4}

Q.2 (a) Answer the following.

1) Prove that if "All men are mortal." and "Socrates is a man." Then "Socrates is a mortal." by using theory of Inference. **04**
2) Determine the truth value of each statement given below. The domain of discourse is the set of real numbers. Justify your answers. **03**
   i) For every x, $x^2 > x$
   ii) For some x, $x^2 > x$
   iii) For every x, if $x > 1$ then $x^2 > x$.

(b) Do the following.

1) Give an example of **04**
   i) A bounded lattice which is complemented but not distributive.
   ii) A bounded lattice which is distributive but not complemented.
   iii) A bounded lattice which is neither distributive nor complemented.
   iv) A bounded lattice which is both distributive and complemented.

2) Two equivalence relations R and S are given by their relation matrices MR and MS. Show that RoS is not an equivalence relation. **03**

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

OR

(b) Define an equivalence relation. Prove that the relation "congruence modulo m" given by ≡ $\{<x, y> / x-y$ is divisible by m$\}$ over the positive integer is an equivalence relation. Also draw the relation graph for this relation using m=5 over the set x={1,2,3,……,10}. **07**

Q.3 (a) Answer the following.

1) Define isomorphic lattices. Draw the Hasse diagrams of lattices **03**

   i) $(S_4 \times S_{25}, D)$ ii) (S_{36}, D)

check whether these lattices are isomorphic?

2) Define complemented lattice. Which of two lattices <Sn, D> for n=30 and n=45 are complemented? Draw Hasse Diagram of these lattices. Are these lattices distributive? Justify your answer. **04**

(b) Answer the following.

1) Define Sub-Boolean Algebra. Find all Sub-Boolean Algebra of $<S_{100}, D>$. **03**

2) For a Boolean Algebra $<B, *, \oplus, ', 0, 1>$ prove that **04**

$(a \oplus b') * (b \oplus c') * (c \oplus a') = (a' \oplus b) * (b' \oplus c) * (c' \oplus a)$

OR

Q.3 **(a)** Answer the following.

1) Use the Quine McClusky method to simplify the SOP expansion, **05**

$F(a,b,c,d) = \Sigma\ (0, 2, 4, 6, 8, 10, 12, 14)$

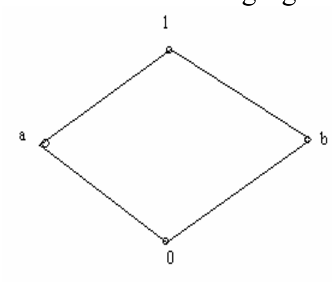
And draw the circuit diagram of the minimized function.

2) In any Boolean algebra, show that **02**

$a \le b \Rightarrow a \oplus (b * c) = b * (a \oplus c)$

(b) Answer the following.

1) Given an expression $\alpha(x1, x2, x3)$ defined to be $\sum(0, 3, 5, 7)$, determine the value of $\alpha(a, b, 1)$, where $a, b, 1 \in B$ and $<B, *, \oplus, ', 0, 1>$ is a Boolean algebra given in the following figure. **04**



2) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean algebra prove the following : **03**

$a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$

Q.4 **(a)** Define "Group" "normal subgroup" "Group homomorphism" of a group. Determine all the proper subgroups of the symmetric group $<S_3, \Diamond>$ given in the table below. Is this group normal? Justify your answer. **07**

◊	P1	P2	P3	P4	P5	P6
P1	P1	P2	P3	P4	P5	P6
P2	P2	P1	P5	P6	P3	P4
P3	P3	P6	P1	P5	P4	P2
P4	P4	P5	P6	P1	P2	P3
P5	P5	P4	P2	P3	P6	P1
P6	P6	P3	P4	P2	P1	P5

(b) Define isomorphic groups. Prove that groups $<Z_5^*, X_5>$ and $<Z_4, +_4>$ are isomorphic, where $Z_5^* = Z_5 - [0]$ **07**

OR

Q.4 **(a)** Define cyclic group. Show that cyclic group is abelian but converse is not true. **07**
Is $<z_5, +_5>$ a cyclic group? If so, find its generators.

(b) Define subgroup of a group, left coset of a subgroup $<H, *>$ in the group $<G, *>$. Find left cosets of $\{[0], [3]\}$ in the group $<Z_6, +_6>$. **07**

Q.5 **(a)** Give an abstract definition of graph. When are two simple graphs said to be isomorphic? Give an example of two simple digraphs having 4 nodes and 4 edges which are not isomorphic. **07**

(b) When a simple digraph is said to be weakly connected, unilaterally connected and strongly connected? Define weak, unilateral and strong components. Write the Strong, unilateral and weak components for the diagraph given in **fig-1**. **07**

OR

Q.5 **(a)** Define nodebase of a simple diagraph. Find the reachability set of all nodes for the diagraph given in **fig-2**. Also find the nodebase for it. **07**

(b) Give three other representations of tree expressed by **07**
(v0(v1(v2)(v3)(v4))(v5(v6)(v7)(v8)(v9))(v10(v11)(v12)))
Obtain binary tree corresponding to it.

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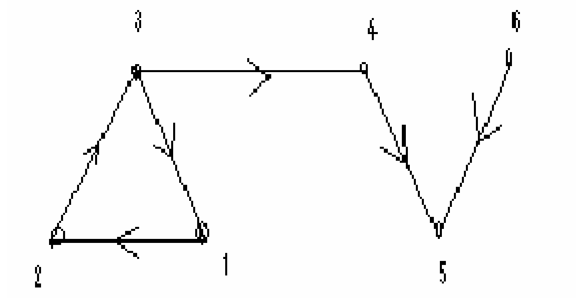

fig-1

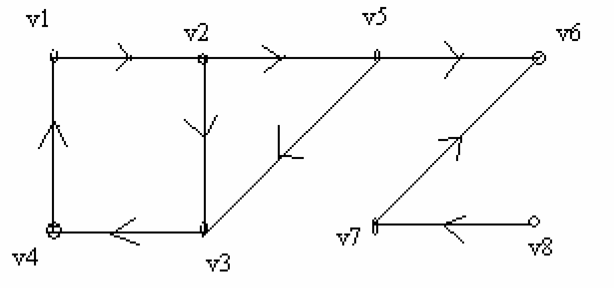

fig-2

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