Seat No.: _________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA SEM-I Examination- Jan.-2012

Subject code: 610003 **Date: 04/01/2012**
Subject Name: Discrete Mathematics for Computer Science (DMCS)
Time: 10.30 am-1.00 pm **Total marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q:1 Do as directed.

(a) 1. Let A= {1, 2, 3} then show that the inclusion relation $\subseteq$ on A is a partial ordering. **03**

2. Define Direct Product and draw the Hasse diagrams of <S,D>, <L,D> and <S X L, D> for S = {1,3,6} and L = {1,2,4}. **03**

3. State the Lagrange's theorem. **01**

(b) 1. Briefly discuss the following terms: **02**
   i) modus pones
   ii) hypothetical syllogisms

2. Explain Isomorphic graphs with example. **04**

3. Write negation of following statement: **01**
   This flower is not beautiful.

Q.2 Do as directed.

(a) 1. Let A = {1,2,3} and R={(x,y)/x<y} then find M_R and construct the digraph of R. **03**

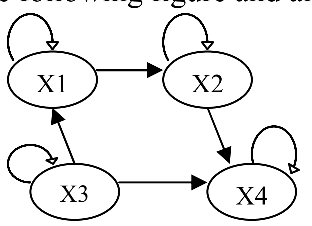
2. Use a truth table to determine whether the following argument form is valid. **02**

   $p \rightarrow q$
   q
   Therefore p

3. Explain abelian group with example. **02**

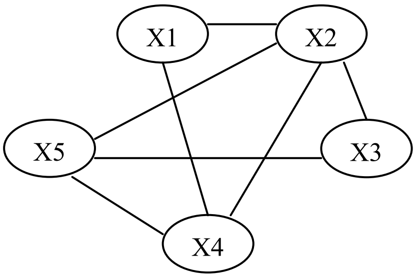
(b) Do as directed.

1. Determine the properties of the relations given by the graph shown in the following figure and also write the corresponding relation matrix. **02**



2. Let S={1,2,3,4} and consider the following collections of subsets of S. Find which subsets are partitions and which subsets are covering of S. **02**
A = {{1,2},{2,3,4}}
B = {{1,2},{3,4}}
C = {{1},{1,2}}

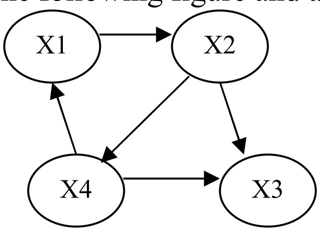
3. Find out maximum compatibility blocks of following digraph and write its relation matrix. **02**



4. Define Poset with example. **01**

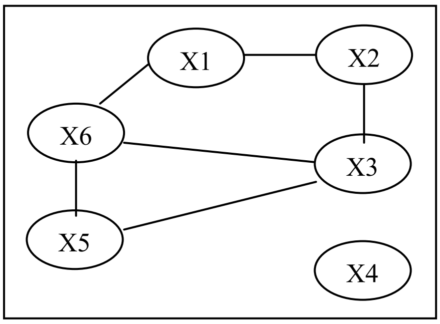
OR

(b) 1. Determine the properties of the relations given by the graph shown in the following figure and also write the corresponding relation matrix. **02**



2. Let S={a,b,c} and consider the following collections of subsets of S. Find which subsets are partitions and which subsets are covering of S. **02**
A = {{a,b},{c}}
B = {{a,b},{b,c}}
C = {{a},{a,b}}

3. Find out maximum compatibility blocks of following digraph and write its relation matrix. **02**



4. Define Equivalence Relation with example. **01**

Q.3 Do as directed.

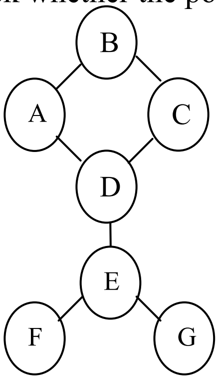
(a) 1. If R and S are equivalence relations on the set X, prove that $R \cap S$ is an equivalence relation. **03**

2. Let R and S be two relations on a set of positive integers I, **02**
R = {<x,2x>/x∈I}
S = {<x,7x>/x∈I}
Then find R 0 S and R 0 R 0 R.

3. Let X = {2,3,6,12,24,36} and the relation ≤ be such that x ≤ y if x divides y. Draw the Hasse Diagram of <x, ≤>. **02**

(b) 1. Given a relation R on the set A = {1,2,3,4,5,6} and the equivalence classes are {1,3,6}, {2} and {4,5} then represent them by using Vector Representation Method. **04**

2. Check whether the poset shown below represents lattice or not? **03**



OR

Q.3 Do as directed.

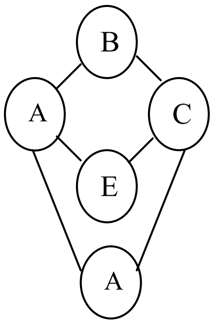
(a) 1. Let R be a relation on N such that aRb if and only if a|b (a divides b) in N. Check whether R is an equivalence relation or not? **03**

2. Let R and S be two relations on a set of positive integers I, **02**
R = {<x,3x>/x∈I}
S = {<x,4x>/x∈I}
Then find R 0 R and R 0 S 0 R.

3. Let S = {2,4,5,10,15,20} and the relation ≤ is the divisibility relation. Draw the Hasse diagram of <S, ≤>. **02**

(b) 1. Show that in a lattice, a ≤ b ⇔ a * b = a ⇔ a ⊕ b = b, where <L, ≤> be a lattice in which * and ⊕ denote the operations of meet and join respectively, for all a,b ∈ L. **04**

2. Check whether the poset shown below represents lattice or not? **03**



Q.4 Do as directed.

(a) 1. Show that < S_{30},*,⊕ > and <P(A), ∩,∪> are isomorphic lattices for A = {a,b,c}. **02**

2. Consider the set B = {0, 1}. Show that <B,*, ⊕,’,0,1> is a Boolean algebra. **02**

3. Write an algorithm COVER which determines the set of prime implicants by using the Quine-McCluskey procedure. **03**

(b) 1. Prove or disprove: If x is an irrational number, then x^2 is irrational. **02**

2. Give an indirect proof to show that if n is an even integer, then n is odd. **02**

3. Prove the following theorem using direct proof method. **03**
For all integer x, if x is odd, then x^2 is odd.

OR

Q:4 Do as directed.

(a) 1. Consider lattice < S_{10}, *, ⊕ >, where S_{10} = {1,2,5,10}. Find the complements of the elements of S_{10}. **02**

2. Write the following Boolean expression in an equivalence sum-of-products canonical form in three variables x1,x2 and x3. **02**
x1 ⊕ x2.

3. Discuss Karnaugh map with example. **03**

(b) 1. Prove without constructing the truth table that $p \wedge (\sim q \rightarrow \sim p) \rightarrow q$ is a tautology **02**

2. Explain biimplication with example **02**

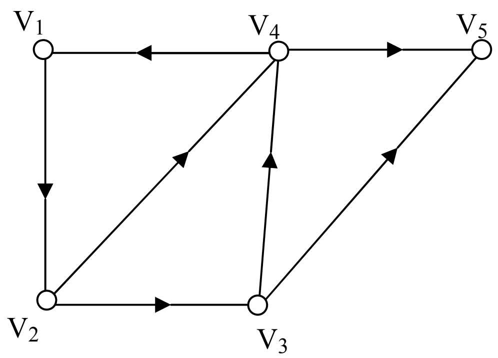
3. Let P(x,y) denote the sentence: 2x + y = 1. What are the truth values of $\forall x \exists y P(x,y)$, $\forall x \forall y P(x,y)$, and $\exists x \exists y P(x,y)$, where the domain of x,y is the set of all integers? **03**

Q.5 **(a)** 1. Explain path and circuit with example. **04**

2. Test the validity of the logical consequence given below: **03**
All birds can fly.
A sparrow is a bird.
Therefore, a sparrow can fly.

(b) 1. Define the following terms: **02**
i) subgroup ii) group homomorphism

2. Find all the indegrees and outdegrees of the nodes of the graph given in following figure. **05**
Give all the elementary cycles of this graph. List all the nodes which are reachable from another node of the diagraph.



OR

Q.5 **(a)** 1. Explain Adjacency matrix of graph G with example. **04**

2. What is a group? List at least two properties of group. **03**

(b) 1. Show that every cyclic group of order n is isomorphic to the group (Z_n, +n) **02**

2. What is ring in group theory? **02**

3. Write a short note on a directed tree **03**

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