**Seat No.: _____** **Enrolment No._____**

# GUJARAT TECHNOLOGICAL UNIVERSITY
## MCA SEM-IV Examination- Dec.-2011

**Subject code: 640003** **Date: 17/12/2011**
**Subject Name: Operations Research (OR)**
**Time: 10.30 am-01.00 pm** **Total marks: 70**

**Instructions:**
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Define Operations Research. Discuss briefly the history of operations research. Also give the application areas of operations research. **05**

**(b)** Obtain the dual of the following primal LP problem : **06**
Minimize $Z = 2x_2 + 5x_3$
subject to the constraints

$$x_1 + x_2 \geq 2$$
$$2x_1 + x_2 + 6x_3 \leq 6$$
$$x_1 - x_2 + 3x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$.

**(c)** Define the following terms : Objective Function, Feasible Solution, Unbounded Solution **03**

**Q.2** **(a)** A firm makes two products X and Y and has a total production capacity of 9 tonnes per day, both X and Y require the same production capacity. The firm has a permanent contract to supply atleast 2 tonnes of X and atleast 3 tonnes of Y per day to another company. Each tonne of X requires 20 machine hours to production time and each tonne of Y requires 50 machine hours of production time. The daily maximum possible number of machine hours is 360. All of the firm's output can be sold. The profit made is Rs. 80/- per tonne of X and Rs. 120/- per tonne of Y. **07**

Formulate this problem as an LP model so as to determine the production schedule which will yield maximum profit (DO NOT SOLVE).

**(b)** Solve the following LP problem using Simplex method : **07**
Maximize $Z = 5x_1 + 3x_2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$

and $x_1, x_2 \geq 0$.

**OR**

**(b)** A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods A and B are available at a cost of Rs. 4/- and Rs. 3/- per unit respectively. If one unit of food A contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 calories, then find by graphical method what combination of foods be used to have least cost. **07**

**P.T.O**

**Q.3** **(a)** Solve the following LP problem using Big-M method : **07**

Maximize $Z = 3x_1 + 2x_2$

subject to the constraints

$$2x_1 + x_2 \leq 2$$
$$3x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$.

**(b)** Given the following pay-off matrix of a 2-person zero sum game, determine the optimal strategies for the players and the value of the game. Is the game strictly determinable ? Is it fair ? **07**

| Player A's strategies | Player B's strategies | | | | |
|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ |
| $A_1$ | -1 | 0 | 0 | 5 | 3 |
| $A_2$ | 3 | 2 | 2 | 2 | 2 |
| $A_3$ | -4 | -3 | 0 | 2 | 6 |
| $A_4$ | 5 | 3 | -4 | 2 | 6 |

**OR**

**Q.3** **(a)** Write the general mathematical model of a Transportation Problem. Determine the initial basic feasible solution to the following transportation problem using Vogel's Approximation Method. **07**

| Sources | Distribution Centers | | | | |
|---|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
| $S_1$ | 2 | 3 | 11 | 7 | 6 |
| $S_2$ | 1 | 0 | 6 | 1 | 1 |
| $S_3$ | 5 | 8 | 15 | 9 | 10 |
| Requirements | 7 | 5 | 3 | 2 | |

**(b)** Explain the following terms : Events, Activities, Looping and Dangling in network analysis. **07**

**Q.4** **(a)** Discuss the various costs involved in an inventory model. **07**

**(b)** Solve the following assignment problem : **07**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| A | 9 | 22 | 58 | 11 | 19 | 27 |
| B | 43 | 78 | 72 | 50 | 63 | 48 |
| C | 41 | 28 | 91 | 37 | 45 | 33 |
| D | 74 | 42 | 27 | 49 | 39 | 32 |
| E | 36 | 11 | 57 | 22 | 25 | 18 |
| F | 3 | 56 | 53 | 31 | 17 | 28 |

**OR**

**Q.4** **(a)** Draw the network diagram for the following activities and find the critical path and total float. **07**

| Job | Job Time (days) | Immediate Predecessors |
|---|---|---|
| A | 13 | — |
| B | 8 | A |
| C | 10 | B |
| D | 9 | C |
| E | 11 | B |
| F | 10 | E |
| G | 8 | D, F |
| H | 6 | E |
| I | 7 | H |
| J | 14 | G, I |
| K | 18 | J |

**P.T.O**

**(b)** An aircraft company uses rivets at a constant rate of 2500 per year. Each unit costs Rs. 30/-. The company personnel estimated that it costs Rs. 130/- to place an order and that the carrying cost of inventory is 10% per year. How frequently should orders be placed ? Also, determine the optimum size of each order. **07**

**Q.5** **(a)** What is the main objective of sequencing problem ? State the assumptions generally made while dealing with sequencing problems. **07**

**(b)** A plant manager is considering the replacement policy for a new machine. He estimates the following costs (all costs in Rs.). **07**

| Year | Replacement cost at the beginning of the year | Resale value at the end of the year | Operating costs |
|---|---|---|---|
| 1 | 100 | 60 | 25 |
| 2 | 110 | 50 | 30 |
| 3 | 125 | 40 | 40 |
| 4 | 140 | 25 | 50 |
| 5 | 160 | 10 | 65 |
| 6 | 190 | 0 | 80 |

Find an optimal replacement policy and its corresponding minimum cost.

**OR**

**Q.5** **(a)** Four jobs 1, 2, 3 and 4 are to be processed on each of the four machines A, B, C and D in the order ABCD. The processing time in minutes are given the following table. Find the minimum total elapsed time when passing is not allowed. Also, find the idle time for each machine. **07**

| Job | : | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Machine A | : | 58 | 30 | 28 | 64 |
| Machine B | : | 14 | 10 | 12 | 16 |
| Machine C | : | 14 | 18 | 16 | 12 |
| Machine D | : | 48 | 32 | 44 | 42 |

**(b)** Customers arrive at a box office window, being manned by a single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find the average waiting time of a customer. Also, determine the average number of customers in the system and the average queue length. **07**

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