Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**MCA- I$^{st}$ SEMESTER–EXAMINATION – MAY/JUNE - 2012**

**Subject code: 2610003 Date: 31/05/2012**

**Subject Name: Discrete Mathematics for Computer Science (DMCS)**

**Time: 02:30 pm – 05:00 pm Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Define: **07**

i) Join irreducible elements.

ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of following Boolean algebra.

i) $(S_{210}, D)$

ii) $\langle P(S), \cap, \cup, ', \Phi, S\rangle$ where S = {a, b, c}

Also draw the Hasse Diagram.

**(b)** Define Lower bound and greatest lower bound. Let P = < 3, 5, 9, 15, 24, 45}, D> be a poset. Draw the Hasse diagram. Find **07**

i) the maximal element. & minimal element.

ii) The greatest and least element.

iii) the lower bounds of {3, 5}, if any & the upper bound of { 9, 15 } , if any

iv) GLB of {15, 45} & LUB of {3, 9, 15}.

**Q.2** **(a)** State the importance & purpose of Discrete Mathematical Structures with its application to computers science. **07**

**(b)** i) Let P(x) be the statement " $x = x^2$ ".If the domain consists of the integers, **02**

what are the truth values of $\forall x$ P(x)and $\exists$ x P(x) **05**

ii) Define: Logical Equivalence of the statement formula. Without constructing truth table show that ( $\neg p \wedge ( \neg q \wedge r)) \vee (q \wedge r) \vee( p \wedge r ) \equiv r$

**OR**

**(b)** i) Define: Disjoint sets. If $A_1 = \{\{1, 2\}.\{3\}\}$, $A_2 = \{\{1\},\{2,3\}\}$ and $A_3 = \{\{1, 2, 3\}\}$, then show that A1, A2, and A3 are mutually disjoint. **03**

ii) Define law of Modus Ponen and Law of Hypothetical Syllogism with an example. **04**

**Q.3** **(a)** i) Define: Equivalence relation. If I be the set of integers and if R be defined **04**

by " a R b iff a – b is an even integer" where a, b ϵ I, then show that the relation R is an equivalence relation.

ii) Define giving example for each term **03**

1. Sublattice
2. Complemented lattice
3. Modular lattice

**(b)** i) Define: Maximal Compatibility Block. Let the compatibility relation on a set {1, 2, 3, 4, 5, 6} be given by following matrix. Construct the graph and find the maximum compatibility blocks **04**

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | | | | |
| 3 | 1 | 1 | | | |
| 4 | 1 | 1 | 1 | | |
| 5 | 0 | 1 | 0 | 0 | |
| 6 | 0 | 0 | 1 | 0 | 1 |
| | 1 | 2 | 3 | 4 | 5 |

ii) State the absorption law for lattice. Verify it for $(S_{45}, D)$ by taking any two elements. **03**

**OR**

**Q.3** **(a)** i) Find the value of Boolean Expression. **04**

$\alpha(x_1, x_2, x_3, x_4) = [x_1 * (x_2 \oplus x_1') * ( x_3 * x_4' * x_2' )] \oplus ( x_1 * x_4)$ where

$x_1 = 5, x_2 = 6, x_3 = 15, x_4 = 3$ in Boolean algebra $<S_{30}$, gcd, lcm, ', 30> and $n' = 30/n$.

**03**

.ii) Prove the Boolean identities

a) $(a * b) \oplus (a * b') = a$

b) $a * (a' \oplus b) = a * b$

**(b)** i) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a,b,c,d)=\Sigma(1,2,5,6, 13, 14, 15)$ **04**

ii) Obtain the sum of product canonical form of Boolean expression in three variables $x_1, x_2, x_3$ for $(x_1 \oplus x_2 ) * x_3$ **03**

**Q.4** **(a)** **07**

Define: Group and Abelian group. Show that in a group (G, $*$), if for a, b $\epsilon$ G, $(a * b)^2 = a^2 * b^2$, then (G, $*$) is an Abelian group. Prove that the set {1, -1, i, -i} form an Abelian multiplicative group (G, x) where i is an imaginary no. $i = \sqrt{-1}$.

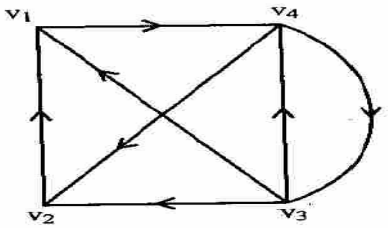
**(b)** Define: Group Homomorphism, Group Isomorphism and Kernel of the homomorphism. Prove that G: $(Z_4, +4) \longrightarrow (Z_5^*, x_5)$ is isomorphism. **07**

**OR**

**Q.4** **(a)** Define Subgroup of a group Find all subgroups of cyclic group of order 12 with generator 'a'. Also find order of generators of G. **07**

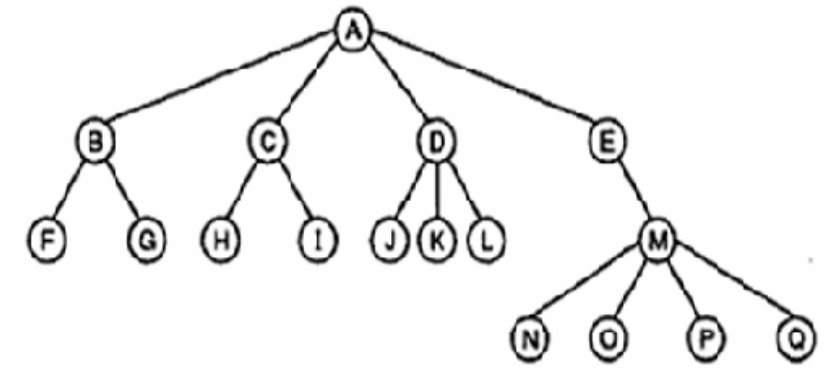
**(b)** Define symmetric group ($S_3$, $\Diamond$). Write composition table of all permutations defined on the symbols 1, 2, & 3 Determine all the proper subgroups of ($S_3$, $\Diamond$). Which subgroup is normal subgroup? **07**

**Q.5** **(a)** Define adjacency matrix of a graph and obtain the adjacency matrix (A) for the following graph. What do transpose of adjacency matrix ( $A^T$) indicate? Draw its graph. State the indegree and outdegree of all the vertices. Find $A^2$ and interpret in detail by stating the results. **07**



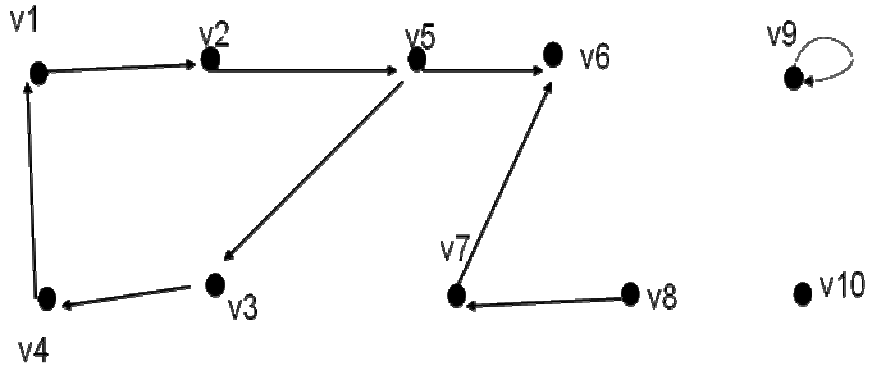

**(b)** i) Define Forest with an example **02**

ii) Define Binary tree. Convert the given tree into the Binary tree. **05**




**OR**

**Q.5** **(a)** Define node base of a diagraph. State its properties. Find all node base of the diagraph given below: **07**




**(b)** Define rooted tree, level of a vertex, leaf, descendents and ancestor of a vertex with a suitable example. Prove that a full m-ary tree with i internal vertex has n = mi +1 vertices **07**

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