Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – SUMMER 2013

Subject Code: 610003 Date: 11-06-2013

Subject Name: Discrete Mathematics for Computer Science

Time: 10:30am to 13:00pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a)1. Define Partial Order Relation and Poset. **02**

2.Define Direct Product of lattices and Draw Hasse Diagrams of <S,D>, <L,D> and <S×L,D> for S={1,3,6} and L={1,2,4} **04**

3. Define Complemented Lattice with example. **01**.

(b) Define Tautology and Contradiction with example. Prove that p -> (p V q) is tautology without constructing truth table. **07**

Q.2 (a)1. Define Group. Check Whether <I, ×> forms a group or not where I is the set of Integers and × is multiplication operation. **03**

2. Define Sum-Of Product canonical form. Write Boolean Expression $x_1 * x_2$ in an equivalent sum-of-product canonical form in three variables x_1, x_2 and x_3. **04**

(b) For an integer prove that the following statements are equivalent: **07**

p: x is divisible by 6.

q: x is divisible by 2 and 3.

r: x is an even number and x is divisible by 3.

OR

(b) Define equivalence relation. **07**

Let Z be the set of integers and R be the relation called Congruence modulo 5" defined by

R = {<x,y> | x ε Z ^ yε Z ^ ($x - y$) is divisible by 5}

Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.

Q.3 (a) Define Cyclic group. Write down the properties of cyclic group. Show that $\langle Z_6, +_6 \rangle$ is a cyclic group of order 6 and also find its generators. **07**

(b) Find a minimal sum-of-product form using K-map **07**

(i) α (x, y, z) = xyz + xyz' + x'yz' + x'y'z

(ii) α (x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z

OR

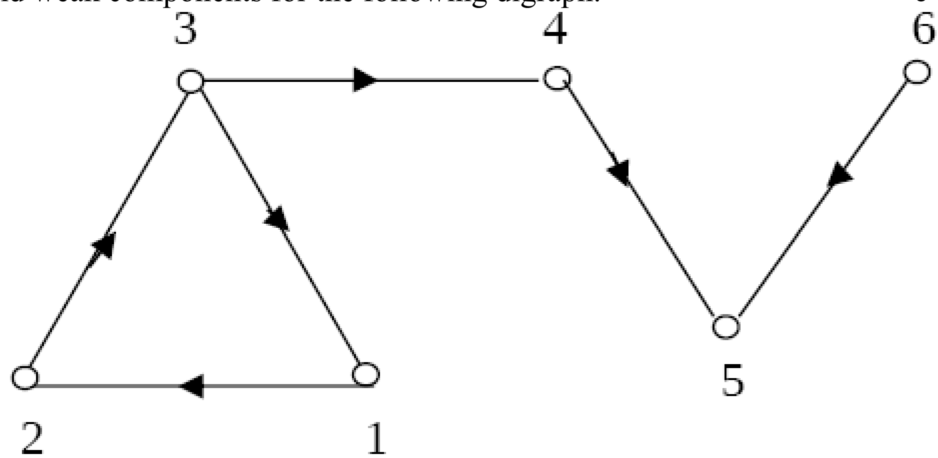
Q.3 (a) Define Sub Boolean Algebra. Find all sub-algebras of Boolean algebra $\langle S_{30}$, *, ⊕, ',0,1>. Write proper steps. **07**

(b)1. Define Join Irreducible element, Meet irreducible element, Atom and Anti atom. **04**

2. Explain Stone's Representation theorem with proper example. **03**

Q.4 **(a)1.** Define weakly connected, unilaterally connected and strongly connected graphs. **03**

2. Define weak, unilateral and strong components. Find the strong, unilateral and weak components for the following digraph. **04**



(b) Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 and 2 between each vertex. **07**

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

OR

Q.4 **(a)** Define Sub-group. Find the subgroups of $<Z_{10}, +_{10}>$ **07**

(b) Define "Lattice as an Algebraic System", "Complete Lattice" and "Distributive Lattice. **07**

Let the sets S0, S1,………………, S7 be given by

S0 = {a, b, c, d, e, f}, S1 = {a, b, c, d, e}, S2 = {a, b, c, e, f}, S3 = {a, b, c, e}, S4 = {a, b, c}, S5 = {a, b}, S6 = {a, c},

S7 = {a}

Draw the diagram of $<L, \subseteq>$,

where L = {S0, S1, S2,………, S7}

Q.5 **(a)** 1. Explain path and circuit with example. **04**

2. Test the validity of the logical consequence given below:

All birds can fly.

A sparrow is a bird.

Therefore, a sparrow can fly. **03**

(b) Find minimal SOP using Quine Mc-cluskey method **07**

F(a,b,c,d) = Σ (0,1,2,5,6,7,8,9,10,14)

OR

Q.5 **(a)1.** Let R be a relation on N such that aRb if and only if a|b (a divides b) in N. Check whether R is an equivalence relation or not? **03**

2. Let R and S be two relations on a set of positive integers I,
R = {<x,3x>/x ε I}
S = {<x,4x>/x ε I}
Then find R 0 R and R 0 S 0 R. **02**

3. Let S = {2,4,5,10,15,20} and the relation ≤ is the divisibility relation.
Draw the Hasse diagram of <S, ≤>. **02**

(b) Define Group Homomorphism. Define isomorphic groups. Prove that groups $<Z_6,+_6>$ is isomorphic to $<Z_7^*,*_7>$, where $Z_7^* = Z_7 - [0]$ **07**

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