Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–II • EXAMINATION – SUMMER 2013

Subject Code: 620005 **Date: 12-06-2013**

Subject Name: Computer Oriented Numerical Methods

Time: 10.30 am - 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Explain total numerical error. How can one control numerical errors ? **07**

(b) State Descartes rule of sign. Use it to determine the number of positive and negative roots of the polynomial equation : $x^4 - 3x^3 + 2x^2 + 20x - 20 = 0$. **04**

(c) Distinguish between open and bracketing methods for finding the roots of an equation $f(x) = 0$. **03**

Q.2 **(a)** Use bisection method to find a root of the equation $x^4 - x - 10 = 0$, in the interval [1,2] correct up to three decimal places. **07**

(b) Use Newton-Raphson method to find a negative root of the following equation $x^3 - 2x^2 + x + 100 = 0$, correct up to three decimal places. Take $x_0 = -1.5$ as the initial guess. **07**

OR

(b) **07**

Find a root of the following equation correct up to three decimal places using secant method : sinö = 1 + ö³.

Q.3 **(a)** From the following table, find y when $x = 1.5$ using Lagrange's interpolation formula. **07**

x :	–2	–1	2	3
y :	–12	–8	3	5

(b) Fit a straight line of the form $y = a + bx$, to the following data : **07**

x :	60	75	100	125	145
y :	225	300	430	560	600

OR

Q.3 **(a)** Given the following values: **07**

x :	40	50	60	70	80	90
y :	184	204	226	250	276	304

Find y when $x = 78$, using Newton's backward interpolation formula.

(b) **07**

Fit a second degree polynomial (parabola) of the form y = aö² + bö + c, to the following data :

x :	–3	–2	–1	0	1	2	3
y :	4.63	2.11	0.67	0.09	0.63	2.15	4.58

Q.4 **(a)** Compute the first order derivative for the following set of data values at **07**

$x = 0.7$ and 0.9.

x :	0.5	0.7	0.9	1.1	1.3	1.5
y :	1.48	1.64	1.78	1.89	1.96	2.00

P.T.O.

(b) Evaluate $\int_0^1 \frac{dx}{1+x}$, using two-point Gauss Legendre Quadrature formula. **07**

OR

Q.4 **(a)** The distance (s) covered by a car in a given time (t) is given in the following table : **07**

Time (minutes) :	10	12	14	16	18
Distance (kms) :	12	15	20	27	37

Find the velocity and acceleration of the car at $t = 18$ minutes.

(b) Find the largest eigen value and the corresponding eigen vector of the following matrix using power method correct upto two decimal places : **07**

$$\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$$

Q.5 **(a)** Solve the following system of simultaneous linear equations using Gauss-Elimination method: **07**

$$x + y - 2z = 3$$
$$4x - 2y + z = 5$$
$$3x - y + 3z = 8$$

(b) Solve the following ordinary differential equation, $dy/dx = x^2 - y$, given that y(0) = 1 and taking step size h = 0.5. Obtain the value of y at x = 1.0 using Runge-Kutta third order method. **07**

OR

Q.5 **(a)** Solve the following system of simultaneous linear equations using Gauss-Seidel method: **07**

$$20x + 2y + 6z = 28$$
$$x + 20y + 9z = -23$$
$$2x - 7y - 20z = -57$$

(b) Solve $\frac{dy}{dx} = x + y^2$, with $y(0) = 1$ for $x = 0.4$ by Adam-Bashforth-Moulton′s Predictor-Corrector method. Find $y(0.1)$, $y(0.2)$ and $y(0.3)$ using Runge-Kutta second order method. **07**

Sr. #	**List of Formulae**
1	$\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
2	$x_0 - \frac{f(x_0)}{f'(x_0)}$
3	$y_k + \Delta y_k u + \frac{\Delta^2 y_k}{2!} u(u-1) + \frac{\Delta^3 y_k}{3!} u(u-1)(u-2) + \ldots + \frac{\Delta^{n-k} y_k}{(n-k)!} u(u-1)\ldots(u-((n-k)-1))$

4	$\frac{1}{\Delta y_0}(y-y_0)$
5	$\frac{1}{\Delta y_0}(y-y_0-\frac{u_1(u_1-1)}{2}\Delta^2 y_0)$
6	$\frac{1}{\Delta y_0}(y-y_0-\frac{u_2(u_2-1)}{2}\Delta^2 y_0-\frac{u_2(u_2-1)(u_2-2)}{6}\Delta^3 y_0)$
7	$(x_{i+1}-x_i)f''(x_i)+2(x_{i+2}-x_i)f''(x_{i+1})+(x_{i+2}-x_{i+1})f''(x_{i+2})=\frac{6[f(x_{i+2})-f(x_{i+1})]}{(x_{i+2}-x_{i+1})}-\frac{6[f(x_{i+1})-f(x_i)]}{(x_{i+1}-x_i)}$
8	$\frac{x_0+x_1}{2}$
9	$\frac{f''(x_i)(x-x_{i+1})^3}{6(x_i-x_{i+1})}+\frac{f''(x_{i+1})(x-x_i)^3}{6(x_{i+1}-x_i)}+\left[\frac{f(x_{i+1})}{x_{i+1}-x_i}-\frac{f''(x_{i+1})(x_{i+1}-x_i)}{6}\right](x-x_i)+$ $\left[\frac{f(x_i)}{x_i-x_{i+1}}-\frac{f''(x_i)(x_i-x_{i+1})}{6}\right](x-x_{i+1})$
10	$y_k+\nabla y_k u+\frac{\nabla^2 y_k}{2!}u(u+1)+\frac{\nabla^3 y_k}{3!}u(u+1)(u+2)+\ldots+\frac{\nabla^{k-1}y_k}{(k-1)!}u(u+1)\ldots(u+((k-1)-1))$
11	$\frac{n\Sigma xy-\Sigma x\Sigma y}{n\Sigma x^2-(\Sigma x)^2}$ $\frac{\Sigma y\Sigma x^2-\Sigma x\Sigma xy}{n\Sigma x^2-(\Sigma x)^2}$
12	$y_k+\Delta_d y_k(x-x_k)+\Delta_d^2 y_k(x-x_k)(x-x_{k+1})+\ldots+\Delta_d^{n-k}y_k(x-x_k)(x-x_{k+1})\ldots(x-x_{n-1})$
13	$\frac{n\Sigma\log x\log y-\Sigma\log x\Sigma\log y}{n\Sigma(\log x)^2-(\Sigma\log x)^2}$ $\frac{\Sigma\log y\Sigma(\log x)^2-\Sigma\log x\Sigma\log x\log y}{n\Sigma(\log x)^2-(\Sigma\log x)^2}$
14	$\frac{n\Sigma x\log y-\Sigma x\Sigma\log y}{n\Sigma x^2-(\Sigma x)^2}$ $\frac{\Sigma\log y\Sigma x^2-\Sigma x\Sigma x\log y}{n\Sigma x^2-(\Sigma x)^2}$
15	$r_0-\frac{b_n}{c_{n-1}}$
16	$f(x_0)+f'(x_0)(x-x_0)+f''(x_0)\frac{(x-x_0)^2}{2!}+\cdots$
17	$T_{n+1}(x)=2xT_n(x)-T_{n-1}(x)$
18	$\frac{\Sigma y}{N}$ $\frac{2}{N}\Sigma y\cos(\omega_0 t)$ $\frac{2}{N}\Sigma y\sin(\omega_0 t)$
19	$\frac{1}{h}\left[\Delta y_k+\frac{\Delta^2 y_k}{2!}(2u-1)+\frac{\Delta^3 y_k}{3!}(3u^2-6u+2)+\frac{\Delta^4 y_k}{4!}(4u^3-18u^2+22u-6)+\ldots\right]$
20	$\frac{1}{h}\left[\nabla y_k+\frac{\nabla^2 y_k}{2!}(2u+1)+\frac{\nabla^3 y_k}{3!}(3u^2+6u+2)+\frac{\nabla^4 y_k}{4!}(4u^3+18u^2+22u+6)+\ldots\right]$
21	$\frac{3h}{8}[y_1+3y_2+3y_3+2y_4+\ldots+3y_n+y_{n+1}]$
22	$\frac{1}{h}\left[\Delta y_k-\frac{\Delta^2 y_k}{2}+\frac{\Delta^3 y_k}{3}-\frac{\Delta^4 y_k}{4}+\ldots\right]$

23	$\frac{1}{h^2}\left[\Delta^2 y_k + \Delta^3 y_k (u-1) + \Delta^4 y_k \frac{6u^2 - 18u + 11}{12} + ...\right]$
24	$\frac{1}{h^2}\left[\Delta^2 y_k - \Delta^3 y_k + \frac{11}{12}\Delta^4 y_k - \frac{5}{6}\Delta^5 y_k + ...\right]$
25	$\frac{h}{3}\left[y_1 + 4y_2 + 2y_3 + 4y_4 + ... + 4y_n + y_{n+1}\right]$
26	$\frac{1}{h}\left[\nabla y_k + \frac{\nabla^2 y_k}{2} + \frac{\nabla^3 y_k}{3} + \frac{\nabla^4 y_k}{4} + ...\right]$
27	$\frac{1}{h^2}\left[\nabla^2 y_k + \nabla^3 y_k (u+1) + \nabla^4 y_k \frac{6u^2 + 18u + 11}{12} + ...\right]$
28	$\frac{1}{h^2}\left[\nabla^2 y_k + \nabla^3 y_k + \frac{11}{12}\nabla^4 y_k + \frac{5}{6}\nabla^5 y_k + ...\right]$
29	$\frac{h}{2}\left[y_1 + 2y_2 + 2y_3 + 2y_4 + ... + 2y_n + y_{n+1}\right]$
30	$y_i + hf(x_i, y_i)$
31	$y_{i+1} = y_i + hs$
32	$s_2 = f(x_i + h, y_i + hs_1)$
33	$s_1 = f(x_i, y_i)$
34	$s_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}s_1)$
35	$s_3 = f(x_i + h, y_i - s_1 h + 2s_2 h)$
36	$s_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}s_2)$
37	$s_4 = f(x_i + h, y_i + s_3 h)$
38	$s = \frac{(s_1 + 2s_2 + 2s_3 + s_4)}{6}$
39	$s = \frac{(s_1 + s_2)}{2}$
40	$s = \frac{(s_1 + 4s_2 + s_3)}{6}$
41	$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_{i-2} - f_{i-1} + 2f_i)$
42	$y_{i+1} = y_i + \frac{h}{24}(f_{i-2} - 5f_{i-1} + 19f_i + 9f_{i+1})$
43	$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i-1} + 4f_i + f_{i+1})$
44	$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$
45	$\frac{b-a}{2}\left[w_1 g(z_1) + w_2 g(z_2)\right]$