Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA SEMESTER –I Regular/Remedial EXAMINATION

Subject code: 2610003 Date: 05/01/2013

Subject Name: Discrete Mathematics for Computer Science

Time: 02:30 pm – 05:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1 (a) Let $X = \{1,2,3,4\}$ and

$R = \{(1,1),(1,4),(4,1),(4,4),(2,2),(2,3),(3,2),(3,3)\}$

(i) write properties of R. [2]

(ii) write the matrix of R. [1]

(iii) Sketch the graph of R.From the graph, write partition of X. [2]

(iv) Is R is a function ? justify your answer. [2]

(b) (i) Define a group. Show that $(Z_6, +_6)$ is a group. [2]

(ii) Define a subgroup of a group. Write all the subgroups of $(Z_6, +_6)$. [3]

What is the relation between order of a subgroup and order of a finite group?

(iii) Define a cyclic group. Is $(Z_6, +_6)$ cyclic? Justify your answer. [2]

Q-2 (a) (I) Give an illustration of [3]

(i) A bounded lattice which is complemented but not distributive.

(ii) A bounded lattice which is distributive but not complemented.

(iii) A bounded lattice which is both distributive and complemented.

(II) Draw Hasse diagram of following Posets . [4]

(i) (S_{30}, D)

(ii) (S_{36}, D)

(b) Define an equivalence relation. Let Z be the set of integers and R be the relation " congruence modulo 5 " defined as [7]

$R = \{(x, y) \ / \ x \in Z \ \wedge \ y \in Z \ \wedge (x - y) \text{ is divisible by } 5 \}$

Show that R is an equivalence relation on Z. Determine the equivalence classes generated by the elements of Z.

OR

(b) (I) Define a partial order relation. Let X be the set of positive integers and R be the relation "xDy " where D stands for "divides". Show that D is a partial order relation on X. [4]

(II) Define a Poset . In Poset (S_{75}, D) find lower bounds of the subsets $A = \{5,15,25\}$ and $B = \{3,15,75\}$. [3]

Q-3 (a) (I) Let p and q be two statements. Define conditional statement $p \to q$ .Write its converse, inverse and contrapositive. [4]

(II) Define universal quantifier. Write the universal quantification of the sentence: $x^2 + x$ is an even integer, where x is an even integer. [3]

Is this universal quantification a true statement ?

(b) (I) Define an abelian group. Show that in a group $(G, \ *)$, if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$ then $(G, \ *)$ must be abelian. [3]

(II) Define: (i) A group homomorphism [4]

(ii) Kernel of homomorphism

Show that kernel of homomorphism g is a subgroup of $(G, \ *)$, where g is a homomorphism from $(G, \ *)$ to (H, Δ).

OR

Q-3 (a) (I) Define existential quantifier. Write existential quantification of the sentence : x is a prime integer where x is an odd integer. [3]

Is this existential quantification a true statement ?

(II) Test the validity of logical consequence : [2]

All dogs fetch.

Ketty does not fetch.

Therefore Ketty is not a dog.

(III) Using truth table, prove that $\sim (p \to q) \equiv p \wedge (\sim q)$. [2]

(b) (I) Define an abelian group. Show that if every element in a group is its own inverse, then the group must be abelian. [3]

(II) Let $(H_1, *)$ and $(H_2, *)$ be subgroups of a group $(G, *)$. Show that $(H_1 \cap H_2, *)$ is also a subgroup of $(G, *)$. Will $(H_1 \cup H_2, *)$ be also a subgroup of $(G, *)$? [4]

Q-4 (a) (I) Define a sublattice. Write any four sublattices of (S_{12}, D). [3]

(II) Define a Boolean Algebra. Show that in a Boolean Algebra $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. [4]

(b) (I) Define a symmetric Boolean expression. Determine whether following expressions are symmetric or not. [3]

(i) $(x_1 * x_2') \oplus (x_1' * x_2)$

(ii) $(x_1' * x_2') \oplus (x_1 \oplus x_2)$

(II) Use the Quine Mc Clusky method to simplify the sum-of-product expression: $f(a,b,c,d) = \sum (10,12,13,14,15)$. [4]

OR

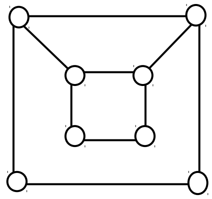
Q-4 (a) Define direct product of two lattices. Show that the lattice (S_{216}, D) is isomorphic to the direct product of lattices (S_8, D) and (S_{27}, D). [7]

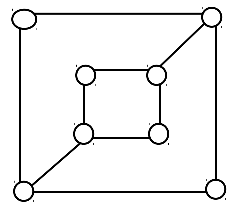
(b) (I) Show that in a Boolean Algebra $a \leq b \Leftrightarrow b' \leq a'$. [3]

(II) Use Karnaugh map representation to find a minimal sum-of-product expression $f(a,b,c,d) = \sum (0,1,2,3,13,15)$. [4]

Q-5 (a) (I) Show that the sum of indegrees of all the nodes of a simple digraph is equal to the sum of outdegrees of all its nodes and that this sum is equal to the number of edges of the graph. [3]

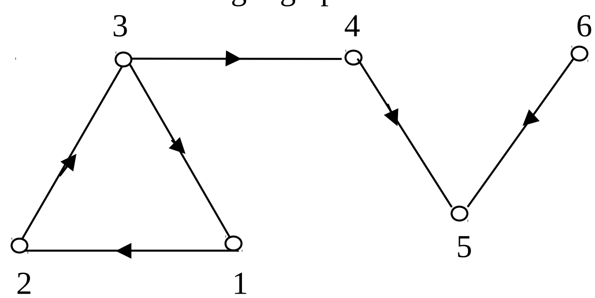
(II) Define isomorphic graphs. State whether the following digraphs are isomorphic or not. Justify your answer. [4]





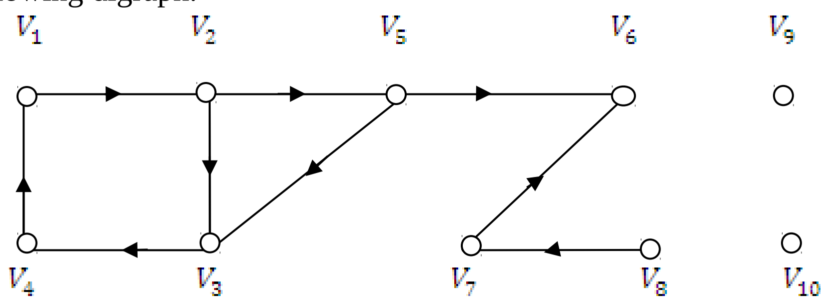
(b) (I) Define weakly connected, unilaterally connected and strongly connected graphs. [3]

(II) Define weak, unilateral and strong components. Find the strong, unilateral and weak components for the following digraph. [4]



OR

Q-5 (a) Define node base of a simple digraph. Find the reachability set of all nodes for the following digraph. [7]



(b) (I) Define (i) A directed tree (ii) A Binary tree (iii) A complete m-ary tree. [3]

(II) Show that in a complete binary tree the total number of edges is given by $2(n_i - 1)$ where n_i is the number of terminal nodes. [4]

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