

GUJARAT TECHNOLOGICAL UNIVERSITY**MCA – SEMESTER - I • EXAMINATION – WINTER 2012****Subject code: 610003****Date: 05/01/2013****Subject Name: Discrete Mathematics for Computer Science (DMCS)****Time: 02:30 pm – 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Prove that $\langle A, *_{17} \rangle$ is group, where $A = \{1, 4, 13, 16\}$ and operation $*_{17}$ represents multiplication modulo 17. **07**

(b) Verify validity of given argument without using truth table: **07**
 “If the meeting will be either in room 302 or 306 then we can find that the meeting is not in 302”.

Q.2 (a) If $A = \{1, 2, 3, 4\}$ and $R = \{ \langle x, y \rangle \mid x = y \}$. Verify! reflexive, symmetric and transitive for given set of relation R. **07**

(b) Define Tautology. Prove that $p \rightarrow (p \vee q)$ is tautology without constructing truth table. **07**

OR**(b)**

(i) Prove that the product of an odd integer and an even integer is even. **03**

(ii) Write rule of Disjunctive Syllogism. Prove it using truth table. **04**

Q.3 (a) Define: Normal-Subgroup. Let $\langle Z_6, +_6 \rangle$ be the group and $H = \{0, 3\}$ be the subgroup of Z_6 . Verify that H is normal subgroup of Z_6 . **07**

(b) Show that (Using laws)

$$(i) [a * (b' \oplus c)]' * [b' \oplus (a * c)']' = a * b * c' \quad \mathbf{04}$$

$$(ii) (x \oplus y) * (x' \oplus y) = y$$

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03**OR**

Q.3 (a) Let the permutations of elements of $\{1, 2, 3, 4, 5, 6\}$ be given by **07**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 4 & 6 \end{bmatrix}$$

Find AB and solve the equation $AX = B$.

(b) Find a minimal sum-of-product form using K-map **07**

$$(i) \alpha(x, y, z) = xyz + xyz' + x'yz' + x'y'z$$

$$(ii) \alpha(x, y, z) = xyz + xyz' + xy'z + x'yz + x'y'z$$

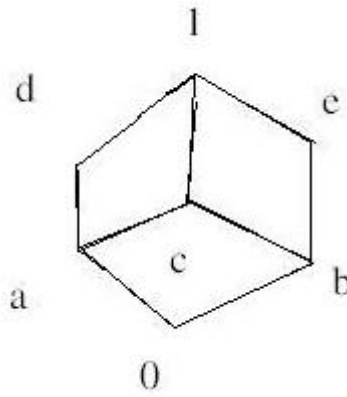
Q.4 (a) Define: Sub-Lattice. Show that S_6 is sub lattice of lattice $\langle S_{30}, D \rangle$. **07**

(b) Draw Hasse diagram and find cover of each element of $\langle L^2, \leq \rangle$, where $\leq$ means $(a, b) R (c, d)$ iff $a \leq c$ and $b \leq d$. L^2 means $L \times L$. $L = \{0, 1\}$ **07**

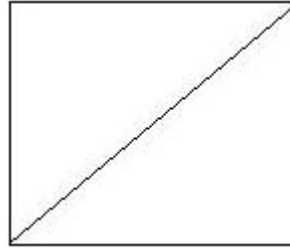
OR

Q.4 (a) Define: Lattice. Prove that, S_8 is lattice and draw Hasse diagram of S_8 . **07**

(b) Define: Join-irreducible, Meet-irreducible, Atoms and Anti-Atom. **07**
 Determine the join-irreducible, meet-irreducible, atoms and anti-atoms for given lattice.



Q.5 (a) Define: Sub Graph. Find all possible sub graphs for the given graph **07**

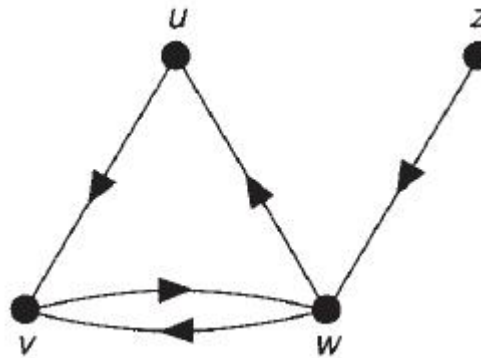


(b) Draw di-graph and find in-degree and out-degree of each vertex from the given adjacency matrix. Using adjacency matrix, find total numbers of path of length 1 and 2 between each vertex. **07**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

OR

Q.5 (a) State handshaking theorem. Verify handshaking theorem for the given graph **07**



(b) Define: Node Base. Find node base for the following graph: **07**

