

GUJARAT TECHNOLOGICAL UNIVERSITY
M.C.A.- SEMESTER – II • EXAMINATION – WINTER 2012

Subject code: 620005**Date: 29-12-2012****Subject Name: Computer Oriented Numerical Methods****Time: 02:30 pm – 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Intermediate calculation steps and results are to be shown, even while using calculator.

- Q.1 (a)** Explain the following terms giving suitable examples : Significant Figures, Absolute Error, Truncation Error, Relative Error. **04**
- (b)** Distinguish between bracketing and open methods for finding the roots of any equation $f(x) = 0$. **04**
- (c)** Obtain the approximate value of $\sin(\pi/12)$ using the Taylor series expansion of the function $\sin(x)$ about $x_0 = 0$. **06**

- Q.2 (a)** State Descarte's rule of sign. Use it to determine the number of positive and negative roots of the polynomial $x^5 - 4x^4 + 5x^3 - 3x^2 - 5x + 9 = 0$. **07**
- (b)** Find the root of the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ using Birge-Vieta method (Hint : Take $r_0 = 3.5$). Perform only three iterations. **07**

OR

- (b)** Use Bisection method to find the root of the following equation $2x - \log_{10}x - 6 = 0$, correct upto three decimal places. **07**

- Q.3 (a)** From the following table, find y when $x = 1.85$ and 1.92 , using appropriate Newton's Interpolation formula. **07**

x :	1.7	1.8	1.9	2.0	2.1	2.2	2.3
y :	5.474	6.050	6.686	7.389	8.166	9.025	9.974

- (b)** Fit an exponential curve, $y = ae^{bx}$, to the following data by the method of least squares. Also, estimate the value of y at $x = 1$. **07**

x :	0	5	8	12	20
y :	3.0	1.5	1.0	0.55	0.18

OR

- Q.3 (a)** From the following table, find y when $x = 0.5$ using Lagrange's interpolation formula. Also, estimate the value of x when $y = 0.3$. **07**

x :	0.4	0.6	0.8
y :	0.3683	0.3332	0.2987

- (b)** It has been observed that the rate of flow of water (x), through a fire engine hose is a quadratic in pressure (y), at the nozzle end. The observed data is : **07**

x :	9.4	11.8	14.7	18.0	23.0
y :	1.0	1.6	2.5	4.0	6.0

Fit a parabola in the form $y = ax^2 + bx + c$ using least squares method.

- Q.4 (a)** From the following table, find the values of the first order derivatives at $x = 53.2$ and 54 , using appropriate Newton's Interpolation formula. **07**

x :	50	51	52	53	54	55
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y	:	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030
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- (b) Evaluate $\int_4^{5.2} \log x dx$ using 07

- (i) Simpson's $\frac{1}{3}$ rule
(ii) Simpson's $\frac{3}{8}$ rule, taking 6 intervals for both cases

OR

- Q.4 (a)** Evaluate the following integrals using two-point Gauss-Legendre Quadrature formula. 07

(i) $\int_0^1 e^{-x^2} \cos x dx$

(ii) $\int_0^1 \frac{dx}{1+x}$

- (b) The following table gives the angular displacements θ (radians) at different intervals of time t (seconds). 07

t	:	0.00	0.02	0.04	0.06	0.08	0.10	0.12
θ	:	0.052	0.105	0.168	0.242	0.327	0.408	0.489

Calculate the angular velocity and acceleration at the instant $t = 0.04$ seconds.

- Q.5 (a)** Find numerically the largest eigen value and the corresponding eigen vector of the following matrix, using the Power method : 07

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

- (b) Given the following differential equation $\frac{dy}{dx} = -2xy^2$, with $y(0) = 1$. Compute $y(0.2)$ and $y(0.4)$ using Runge-Kutta fourth order method. 07

OR

- Q.5 (a)** Explain the pitfalls of Gauss Elimination methods. Use Gauss Elimination method to solve the following system of simultaneous linear equations. 07

$$\begin{aligned} x + 4y - z &= -5 \\ x + y - 6z &= -12 \\ 3x - y - z &= 4 \end{aligned}$$

- (b) Given the following differential equation $\frac{dy}{dx} = y - \frac{2x}{y}$, with $y(0) = 1$. Compute $y(0.1)$, $y(0.2)$ and $y(0.3)$ using Euler's method and obtain $y(0.4)$ using Milne-Simpson's predictor corrector method. 07

Sr. #	List of Formulae
1	$\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
2	$x_0 - \frac{f(x_0)}{f'(x_0)}$
3	$y_k + \Delta y_k u + \frac{\Delta^2 y_k}{2!} u(u-1) + \frac{\Delta^3 y_k}{3!} u(u-1)(u-2) + \dots + \frac{\Delta^{n-k} y_k}{(n-k)!} u(u-1)\dots(u-((n-k)-1))$

4	$\frac{1}{\Delta y_0}(y - y_0)$
5	$\frac{1}{\Delta y_0}(y - y_0 - \frac{u_1(u_1 - 1)}{2}\Delta^2 y_0)$
6	$\frac{1}{\Delta y_0}(y - y_0 - \frac{u_2(u_2 - 1)}{2}\Delta^2 y_0 - \frac{u_2(u_2 - 1)(u_2 - 2)}{6}\Delta^3 y_0)$
7	$(x_{i+1} - x_i)f''(x_i) + 2(x_{i+2} - x_i)f''(x_{i+1}) + (x_{i+2} - x_{i+1})f''(x_{i+2}) = \frac{6[f(x_{i+2}) - f(x_{i+1})]}{(x_{i+2} - x_{i+1})} - \frac{6[f(x_{i+1}) - f(x_i)]}{(x_{i+1} - x_i)}$
8	$\frac{x_0 + x_1}{2}$
9	$\frac{f''(x_i)(x - x_{i+1})^3}{6(x_i - x_{i+1})} + \frac{f''(x_{i+1})(x - x_i)^3}{6(x_{i+1} - x_i)} + \left[\frac{f(x_{i+1})}{x_{i+1} - x_i} - \frac{f''(x_{i+1})(x_{i+1} - x_i)}{6} \right] (x - x_i) +$ $\left[\frac{f(x_i)}{x_i - x_{i+1}} - \frac{f''(x_i)(x_i - x_{i+1})}{6} \right] (x - x_{i+1})$
10	$y_k + \nabla y_k u + \frac{\nabla^2 y_k}{2!} u(u+1) + \frac{\nabla^3 y_k}{3!} u(u+1)(u+2) + \dots + \frac{\nabla^{k-1} y_k}{(k-1)!} u(u+1)\dots(u + ((k-1) - 1))$
11	$\frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \quad \frac{\Sigma y\Sigma x^2 - \Sigma x\Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}$
12	$y_k + \Delta_d y_k (x - x_k) + \Delta_d^2 y_k (x - x_k)(x - x_{k+1}) + \dots + \Delta_d^{n-k} y_k (x - x_k)(x - x_{k+1})\dots(x - x_{n-1})$
13	$\frac{n\Sigma \log x \log y - \Sigma \log x \Sigma \log y}{n\Sigma (\log x)^2 - (\Sigma \log x)^2} \quad \frac{\Sigma \log y \Sigma (\log x)^2 - \Sigma \log x \Sigma \log x \log y}{n\Sigma (\log x)^2 - (\Sigma \log x)^2}$
14	$\frac{n\Sigma x \log y - \Sigma x \Sigma \log y}{n\Sigma x^2 - (\Sigma x)^2} \quad \frac{\Sigma \log y \Sigma x^2 - \Sigma x \Sigma x \log y}{n\Sigma x^2 - (\Sigma x)^2}$
15	$r_0 - \frac{b_n}{c_{n-1}}$
16	$f(x_0) + f'(x_0)(x - x_0) + f''(x_0)\frac{(x - x_0)^2}{2!} + \dots$
17	$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$
18	$\frac{\Sigma y}{N} \quad \frac{2}{N} \Sigma y \cos(\omega_0 t) \quad \frac{2}{N} \Sigma y \sin(\omega_0 t)$
19	$\frac{1}{h} \left[\Delta y_k + \frac{\Delta^2 y_k}{2!} (2u - 1) + \frac{\Delta^3 y_k}{3!} (3u^2 - 6u + 2) + \frac{\Delta^4 y_k}{4!} (4u^3 - 18u^2 + 22u - 6) + \dots \right]$
20	$\frac{1}{h} \left[\nabla y_k + \frac{\nabla^2 y_k}{2!} (2u + 1) + \frac{\nabla^3 y_k}{3!} (3u^2 + 6u + 2) + \frac{\nabla^4 y_k}{4!} (4u^3 + 18u^2 + 22u + 6) + \dots \right]$
21	$\frac{3h}{8} [y_1 + 3y_2 + 3y_3 + 2y_4 + \dots + 3y_n + y_{n+1}]$
22	$\frac{1}{h} \left[\Delta y_k - \frac{\Delta^2 y_k}{2} + \frac{\Delta^3 y_k}{3} - \frac{\Delta^4 y_k}{4} + \dots \right]$
23	$\frac{1}{h^2} \left[\Delta^2 y_k + \Delta^3 y_k (u - 1) + \Delta^4 y_k \frac{6u^2 - 18u + 11}{12} + \dots \right]$
24	$\frac{1}{h^2} \left[\Delta^2 y_k - \Delta^3 y_k + \frac{11}{12} \Delta^4 y_k - \frac{5}{6} \Delta^5 y_k + \dots \right]$

25	$\frac{h}{3}[y_1 + 4y_2 + 2y_3 + 4y_4 + \dots + 4y_n + y_{n+1}]$
26	$\frac{1}{h}\left[\nabla y_k + \frac{\nabla^2 y_k}{2} + \frac{\nabla^3 y_k}{3} + \frac{\nabla^4 y_k}{4} + \dots\right]$
27	$\frac{1}{h^2}\left[\nabla^2 y_k + \nabla^3 y_k(u+1) + \nabla^4 y_k \frac{6u^2 + 18u + 11}{12} + \dots\right]$
28	$\frac{1}{h^2}\left[\nabla^2 y_k + \nabla^3 y_k + \frac{11}{12}\nabla^4 y_k + \frac{5}{6}\nabla^5 y_k + \dots\right]$
29	$\frac{h}{2}[y_1 + 2y_2 + 2y_3 + 2y_4 + \dots + 2y_n + y_{n+1}]$
30	$y_i + hf(x_i, y_i)$
31	$y_{i+1} = y_i + hs$
32	$s_2 = f(x_i + h, y_i + hs_1)$
33	$s_1 = f(x_i, y_i)$
34	$s_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}s_1)$
35	$s_3 = f(x_i + h, y_i - s_1h + 2s_2h)$
36	$s_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}s_2)$
37	$s_4 = f(x_i + h, y_i + s_3h)$
38	$s = \frac{(s_1 + 2s_2 + 2s_3 + s_4)}{6}$
39	$s = \frac{(s_1 + s_2)}{2}$
40	$s = \frac{(s_1 + 4s_2 + s_3)}{6}$
41	$y_{i+1} = y_{i-3} + \frac{4h}{3}(2f_{i-2} - f_{i-1} + 2f_i)$
42	$y_{i+1} = y_i + \frac{h}{24}(f_{i-2} - 5f_{i-1} + 19f_i + 9f_{i+1})$
43	$y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i-1} + 4f_i + f_{i+1})$
44	$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$
45	$\frac{b-a}{2}[w_1g(z_1) + w_2g(z_2)]$

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