Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER–I • EXAMINATION – WINTER • 2014

Subject Code: 2610003 Date: 30-12-2014

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10:30 am - 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (I) Given $S=\{1,2,3,4,5\}$ and a relation R on S where $R=\{(x,y)/x+y=5\}$, What are the properties of the relation R? **03**

(II) Let $X=\{1,2,3,4\}$ and $R=\{(x,y)/x>y\}$ **04**

(i) Draw graph of R. (ii) Write its matrix.

(b) (I) Define a group. Show that $(I,+)$, where I is the set of integers and + is Operation of usual addition, is a group. **03**

(II) Define a subgroup. Show that $\langle \{[1],[4],[13],[16]\}, \times_{17} \rangle$ is a subgroup of $\langle Z_{17}^{*}, \times_{17} \rangle$ **04**

Q.2 **(a)** (I) State whether following lattices are distributive as well as complemented or not. Justify your answers. **03**

(i) (S_{16}, D) (ii) (S_{45}, D)

(II) Draw Hasse diagram of following Posets. **04**

(i) $(\rho(A), \subseteq)$ where $A=\{a,b,c\}$

(ii) (S_{36}, D)

(b) (I) Let $X=\{1,2,3,4,5,6,7\}$ and $R=\{(x,y)/x-y$ is divisible 3$\}$ Show that R is an equivalence relation. **04**

(II) Give an example of a set X such that $\langle \rho(X), \subseteq \rangle$ is a chain. **03**

OR

(b) (I) Given a set $S=\{1, 2, 3, 4, 5\}$. Find the equivalence relation on S, which generates the partition $\{\{1,2\}, \{3\}, \{4,5\}\}$. **03**

(II) Let $R=\{(1,2), (3,4), (2,2)\}$ and $S=\{(4,2), (2,5), (3,1), (1,3)\}$, Find $R \circ S$, $S \circ R$, $R \circ R$, $S \circ S$. **04**

Q.3 **(a)** (I) Determine the truth value of each of the following statements. **03**

(i) 72 > 15 and 33 is a prime integer.

(ii) 19 – 4 = 15 or today's temperature is below freezing.

(iii) If Mickey is in Florida, then 17 is an odd integer.

(II) Test the validity of the logical consequences: **04**

All dogs fetch.

Kitty does not fetch.

Therefore Kitty is not a dog.

(b) (I) Define a cyclic group. Show that $(Z_6, +_6)$ is a cyclic group. **02**

(II) Define (i) A group homomorphism (ii) A group isomorphism. Show that $(Z_4, +_4)$ is isomorphic to $(Z_5^{*}, \times_5)$ **05**

OR

Q.3 **(a)** (I) Show by truth table that the following statement formula is a tautology: **03**

$((p \to q) \wedge (q \to r)) \to (p \to r)$

(II) Test the validity of the logical consequences: **04**
All men are mortal.
Randy is a man.
Therefore Randy is mortal.

(b) (I) Define an abelian group. Show that in a group $(G,*)$, if for any $a,b\in G,\ (a*b)^2=a^2*b^2$ then $(G,*)$ must be abelian. **03**

(II) Define kernel of a group homomorphism. Show that the kernel of Homomorphism g is a subgroup of $(G,*)$, where g is a homomorphism from $(G,*)$ to (H,Δ). **04**

Q.4 (a) (I) Show that in a lattice $(L,\leq)$, for any $a,b\in L,\ a\leq b \Leftrightarrow a\oplus b=b$ **03**

(II) Define a sublattice. Find all sublattices of (S_{10},D) **04**

(b) (I) Show that the following Boolean expressions are equivalent. Obtain their sum-of-products canonical form. **03**

$(i)\ (x\oplus y)*(x'\oplus z)*(y\oplus z)$

$(ii)\ (x\oplus y)*(x'\oplus z)$

(II) Use the Quine - McCluskey method to simplify the sum-of-products expression: **04**

$f(a,b,c,d)=\sum(10,12,13,14,15)$

OR

Q.4 (a) (I) Show that in a lattice $(L,\leq)$, for any $a,\ b\in L,\ a\leq b \Leftrightarrow a*b=a$ **03**

(II) Define a Boolean Algebra. Show that in a Boolean Algebra, **04**
$(a*b)'=a'\oplus b'$ and $(a\oplus b)'=a'*b'$

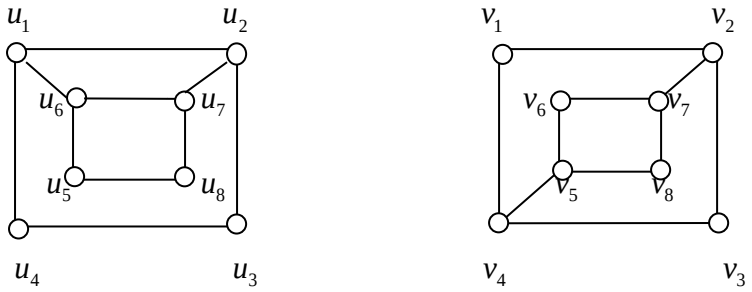
(b) (I) Define (i) A join-irreducible element (ii) A meet-irreducible element in a Boolean Algebra. Write all join-irreducible elements and meet-irreducible elements of (S_{30},D). **04**

(II) Use the Karnaugh map representation to find a minimal sum-of-products expression of following function. **03**

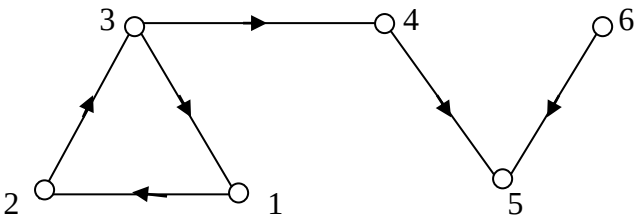
$f(a,b,c,d)=\sum(0,1,2,3,12,14)$

Q.5 (a) (I) Define a path in graph. Define length of the path. What is difference between a simple path and an elementary path? **03**

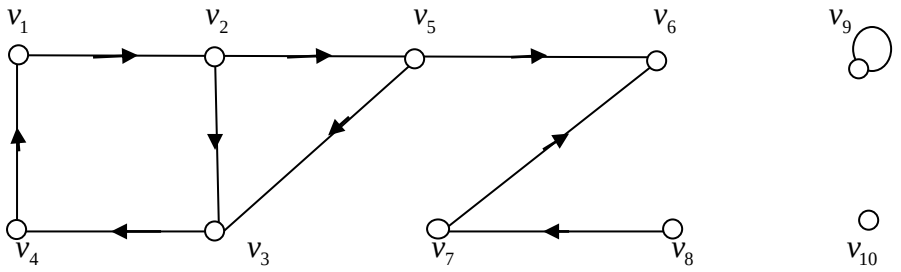
(II) Define isomorphic graphs. State whether the following graphs are isomorphic or not. **04**



(b) (I) Define a strong component. Write strong components of the digraph given in following figure. **03**

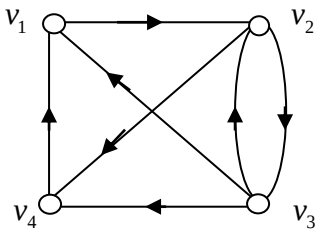


(II) Define a node base. Write node base of the digraph given in following digraph. **04**

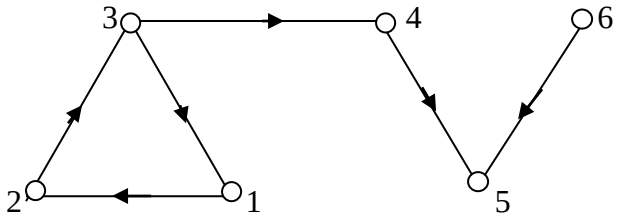


OR

Q.5 **(a)** (I) Define adjacency matrix of the digraph *G*. Write adjacency matrix of the following digraph. **04**



(II) Define a unilateral component. Write unilateral components of the Graph given in following figure. **03**



(b) (I) Define a complete binary tree. What will be number of terminal nodes In a complete binary tree having 8 edges? **04**

(II) Define (i) An $m-ary$ tree.(ii) An ordered tree (iii) A positional $m-ary$ **03**

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