Seat No.: _________ Enrolment No._______________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA SEM-I Examination- Jan.-2012

Subject code: 2610003 **Date: 04/01/2012**

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10.30 am-1.00 pm **Total marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

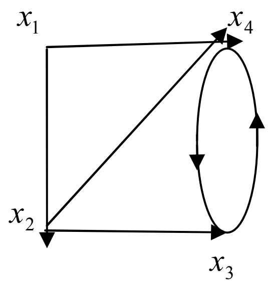
Q.1	**(a)**	Define abelian group and cyclic group. Prove that the set of permutations on the set S={x,y} is a group under the operation ◊ of composition of permutations. Is it an abelian group?	**02** **05**
	(b)	(1)Represent the set P={a, e, i, o, u} in set builder form.	**01**
		(2)Find the power set of the set Q={1, {2, 3}, 4 }	**02**
		(3)For A={2, 3, 4, 5, 6}, B={3, 4, 5, 6, 7}, C={4, 5, 6, 7, 8} find (i) $(A\cup B)\cap(A\cup C)$ (ii) $(A\cap B)\cup(A\cap C)$	**04**
Q.2	**(a)**	Draw hasse diagram for the following.	**03**
		(i) (S_{60}, D) (ii) $(S_4 \times S_3, \le)$. Here S_n is the set of devisors of n.	**04**
	(b)	Define order of an element of a group, giving an example. Prove that $(Z_6, +_6)$ is isomorphic to $(Z_7^*, \times_7)$, where $Z_7^* = Z_7 - \{\overline{0}\}$.	**02** **05**
		OR	
	(b)	Define subgroup of a group. Determine all subgroups of the group $(G, \times)$, where G={-1, 1, -i, i}.	**01** **06**
Q.3	**(a)**	(1) On the set N of natural numbers the relation R is defined as xRy if x>y. Check whether or not R is reflexive, symmetric and transitive.	**03**
		(2)Construct the truth table for the following formula. $[(\neg Q) \wedge (P \Rightarrow Q)] \Leftrightarrow P$	**04**
	(b)	Define giving an example universal quantifier and existential quantifier. Symbolize the following statements by using quantifiers, predicates and logical connectives.	**02**
		(1)Any finite set with n elements has 2^n subsets.	**03**
		(2)All integers are rational numbers.	**02**
		OR	
Q.3	**(a)**	Name different techniques of proof. Explain "the method of proof by contradiction", giving suitable example.	**02** **05**
	(b)	Prove that (S_{66}, D) is a Boolean algebra.	**07**
Q.4	**(a)**	(i) Show that in a lattice if $x \le y \le z$ then $x \oplus y = y * z$	**03**
		(ii)In a lattice show that $(a*b)\oplus(c*d) \le (a\oplus c)*(b\oplus d)$.	**04**
	(b)	In any Boolean algebra prove that (i) $a = 0 \Leftrightarrow ab' + a'b = b$ (ii) $(a+b)' = a'b'$ (iii) $(ab)' = a' + b'$	**03** **03** **01**

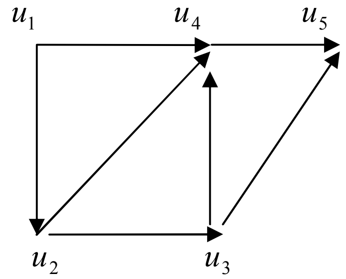
OR

Q.4 **(a)** Define sub-lattice of a lattice. Determine which of the following is a sub-lattice of $(S_{30}, *, \oplus)$, (i) $S_1 = \{1,2,3,6\}$ (ii) $S_2 = \{1,3,6,15\}$. **01** **03** **03**

(b) Obtain the values of the Boolean forms (i) $x_1 * x_2$ (ii) $x_1 \oplus (x_1 * x_2)$ (iii) $x_1 * (x_1' \oplus x_2)$ over the ordered pairs of the Boolean algebra B= {0, 1}. **02** **05**

Q.5 **(a)** Define node base. Find the node bases for each of the following digraphs. **01** **03** **03**



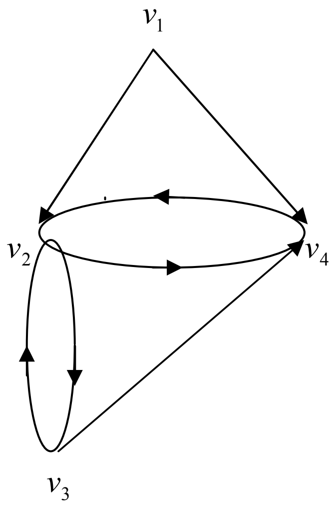


(b) Write the following Boolean expressions in an equivalent Sum Of Products (SOP) canonical form in three variables x_1, x_2 *and* x_3. **03** **04**

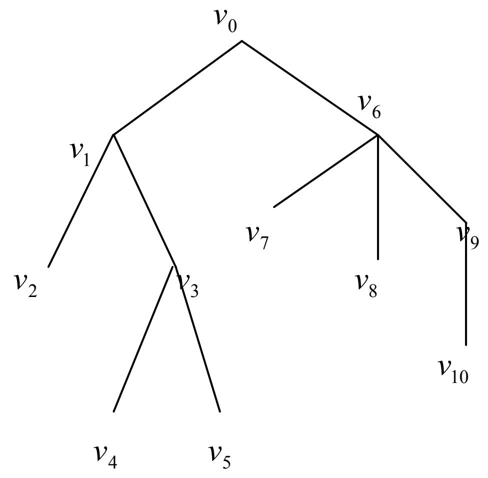
(i) $x_1 * x_2$ (ii) $(x_1 \oplus x_2)' * x_3$

OR

Q.5 **(a)** Define adjacency matrix of a digraph. Obtain the adjacency matrix A of the given digraph. Find the elementary paths of lengths 1 and 2 from v_1 to v_4. **01** **03** **03**



(b) Define directed tree. Draw different representations of the following tree. **01** **06**



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