

GUJARAT TECHNOLOGICAL UNIVERSITY**M.E Sem-I Examination January 2010****Subject code:711305 Subject : Statistics and Optimization Techniques****Date: 27 / 01 / 2010****Time: 12.00 – 2.30 pm****Total Marks: 60****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) The population in a community for the year 1962 to 1972 is tabulated as follows **06**

Year	Population
1962	24000
1963	24540
1964	24750
1965	25100
1966	25340
1967	25820
1968	26100
1969	26200
1970	26500
1971	26800
1972	27450

It is suggested that the population in a given year will be depend on the population of the previous year as predicted from the following model $x_i = a + bx_{i-1}$ where x_i and x_{i-1} are the population of i th and $(i-1)$ th year respectively, Based on the population data , determine the estimates for a, and b,. Based on the model predict the population in 1973. Also find out the cofficeint of regression model.

(b) Solve the following LPP by using Graphical Method **06**Maximize $Z = -4x_1 + 3x_2$ Subject to , $x_1 - x_2 \leq 0$ $x_1 \leq 4$ $x_1, x_2 \geq 0$ **Q.2 (a)** 1. Define Probability. Explain Poission process.. **06**

2. Check wether the following can define PDF and explain your answer

i.) $f(x) = \frac{(5-x^2)}{6}$ for $x = 1, 2, 3$

ii.) $f(x) = \frac{k}{2^x}$ for $x = 1, 2, 3, \text{ and } 4$

(b) Calculate probabilities from the probabilyty density function. If a random variable **06**

has the probability density $f(x) = 2e^{-2x}$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$. Find the probabilyty that will take on a value (a) 1 and 3, (b) greater than 0.5

OR

- (b) 1. Write note on Central limit theory. 06
 2. What is conditional probability? Define its rules.

- Q.3 (a)** 1. Considering a 100 km. highway and assume that road condition and traffic volume are uniform throughout the 100km distance so that accidents are equally likely to occur any where on the highway. Define the event
 A= An accident in 0 km. to 30 km.
 B= An accident in 20 km. to 60 km
 Since accidents are equally likely anywhere along accidents occur in the interval (20,60) What are the probability of events A and B. 06

2. For the Purpose of designing a left turn bay at highway intersection, the left turns of vehicles may be modeled as poisson process. If the cycle time of the traffic light for left turn is 1 minutes and the design criterion requiries left turn lane that will sufficient 96% of the time , what should the lane distance (in term of car lenth) be to allow for an average of 100 left turn per hour?

- (b) 1. If the random variable has the standard normal distribution, find the probability that it will taken on a value
 (a) less than 1.5 (b) less than -1.2 (c) greater than 2.16 06
 2. The air pollution in a city is caused mainly by industrial and automobile exhausts. In the next 5 years the chances of successfully controlling these two sources of polution are respectiely 75% and 60% Assume that if only one of the two sources is sucessfully controlled , the probability of bringing the pollution below acceptance level would be 80% (a) what is the probability of successfully controlling air pollution in the next 5 years.(b)if in the next five year the pollution level is not sufficiently controlled what is the probability that it is entirely caused by the failure to control automobile exhaust?

OR

- Q.3 (a)** For the purpose of designing the left-turn lane for east bound traffic, the 60 observations are made random of the numbers of car waiting for left turns at intersection yielded the following results. 06

No of Cars	No. of Observation
0	4
1	16
2	20
3	14
4	3
5	2
6	1
7	0
8	0

E1=Event more than 2 cars weighting for left turn
 E2=Event that 2 to 4
 Find the probability of E1,E2, $E1 \cap E2$ and $E1 \cup E2$

- (b) 1. Explain Binomial distribution. State its assumption. How does the Hypergeometric distribution differ from Binomial distribution? **06**
2. With reference to the delivery of 12 new cars containing two red color cars. How many ways one can choose three cars of this and get
- None of the car is of red color
 - One of the car is of red color
 - Both of the red color cars

- Q.4 (a)** Use Simplex method to solve the following LPP **06**
- Maximize $Z = 2x_1 + 5x_2$
 Subject to , $x_1 + 4x_2 \leq 24$
 $3x_1 + x_2 \leq 21$
 $x_1 + x_2 \leq 9$
 $x_1, x_2 \geq 0$

- (b) Define: Jockeying, Balking and Reneging for Queueing Theory. Also Write about Empirical Queueing models. **06**

OR

- Q.4 (a)** A firm can produce three types of cloths A, B and C. three kinds of wool are required for it say, Red Wool, Green Wool, Blue Wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards of blue wool; one unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool, and 1 unit length of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has a stock of only 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3, of type B cloth is Rs. 5, and that of type C cloth is Rs. 4. formulate the problem as LPP. **06**

- (b) Use Simplex method to solve the following LPP **06**
- Maximize $Z = 10x_1 + 15x_2 + 20x_3$
 Subject to , $2x_1 + 4x_2 + 6x_3 \leq 24$
 $3x_1 + 9x_2 + 6x_3 \leq 30$
 $x_1, x_2, x_3 \geq 0$

- Q.5 (a)** The arrival rate of customers at the single window booking counter of a 2 Wheeler agency follows poisson distribution and the service time follows exponential distribution. The service rate follows poisson distribution. The arrival rate and the service rate are 25 customers per hour and 35 customers per hour respectively. **06**

Find the following :

- Utilization of the booking clerk
- Average no. of waiting customers in the Queue.
- Average no. of waiting customers in the System
- Average waiting time per customer in the Queue.

- (b) Find initial feasible solution by Vogel's approximate method. **06**
- F : Factory, W : Warehouse

	W1	W2	W3	W4	Capacity
F1	10	30	50	10	07
F2	70	30	40	60	09
F3	40	08	70	20	18
Requirement	05	08	07	14	

OR

- Q.5 (a)** Discuss zero one programming model for assignment problem: Enlist the methods to solve assignment problem. **06**

(b) Find initial feasible solution by

(a) Min.Cost method

(b) N-W corner method

	M1	M2	M3	M4	M5	Supply
P1	10	02	16	14	10	300
P2	06	18	12	13	16	500
P3	08	04	14	12	10	825
P4	14	22	20	08	18	375
Demand	350	400	250	150	400	

P : Plant , M : Market
