

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code: 711403N

Subject Name: Statistical and Numerical Analysis

Date: 02 /02 /2011

Time: 02.30 pm – 05.00 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** What is called an interpolating polynomial? Established the uniqueness of interpolating polynomial. Find the Lagrange interpolating polynomial that fits the following data: 07

x	-1	1	4	7
$f(x)$	-2	0	63	342

Hence, find approximately $f(5.0)$

- (b)** Show that the matrix $\begin{bmatrix} 2 & 1 & -2 \\ 4 & 2 & 3 \\ -6 & 3 & 7 \end{bmatrix}$ is non-singular, but it cannot be written as the product of the lower and the upper triangular matrices. 07

- Q.2 (a)** State the two-point Gauss-Laguerre formula to find $\int_0^\infty e^{-x} f(x) dx$. Show that its error constant is 4. Also, evaluate $\int_0^\infty e^{-x} \sin x dx$, approximately, using the formula. 07

- (b)** Solve the system of equations 07

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

using the Gauss - Seidal with error format, taking initial approximation $\bar{x}^{(0)} = (0, 0, 0)$ and perform three iteration.

OR

- (b)** State the nodes and weight for the two - point Gauss - Chebyshev formula. Also, evaluate $\int_{-1}^1 (1 - x^2)^{3/2} \cos x dx$, using the same. 07

- Q.3 (a)** Find the inverse of the matrix $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$; using the Cholesky's method. 07

Also, determine the condition number of the matrix.

- (b)** (i) In a group of 160 graduate engineering students, 92 are enrolled in an advanced course in statistics, 63 are enrolled in a course in operations research, and 40 are enrolled in both. How many of these students are not enrolled in either course? 07
- (ii) A car rental agency has 18 compact cars and 12 intermediate size cars. If four of the cars are randomly selected for a safety check, what is the probability of getting two of each kind?

OR

Q.3 (a) Evaluate $\int_0^1 (1 + \frac{\sin x}{x}) dx$, correct to 3 - decimal places, using trapezoidal rule and Romberg integration, where $f(0)$ is taken as the limit value. **07**

(b) If a card is drawn from a well shuffled deck of 52 cards, what is the probability of drawing
(i) A black card, (ii) A heart card (iii) A queen (iv) A jack of diamond. **07**

Q.4 (a) A distributor makes a profit of ` 20 on an item if it is shipped from the factory in perfect condition and arrives on time, but it is reduced by ` 2 if it does not arrive on time, and by ` 12 regardless of whether it arrives on time if it is not shipped from the factory in perfect condition. If 70% of such items are shipped in perfect conditions and arrive on time 10% are shipped in perfect condition but do not arrive on time, and 20% are not shipped in perfect condition, what is the distributors expected profit per item. **07**

(b) Four people make repairs of breakdowns. Jack who services 20% of breakdowns, makes an incomplete repair 1 time in 20, Tom who services 60% of the break downs, makes an incomplete repair 1 time in 10, George who services 15% of the breakdowns makes an incomplete repair 1 time in 10, and Peter who services 5% of the breakdowns makes an incomplete repair 1 time in 20. For the next problem with the production line diagnosed as being due to an initial repair that was incomplete what is the probability that this initial repair was made by Jack? **07**

OR

Q.4 (a) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of the chips receive a thick enough coating, find the probabilities that among 8 chips **07**

- (i) Exactly 6 will have thick enough coating
- (ii) At least 2 will have thick enough coating
- (iii) At most 2 will have thick enough coating.

Q.4 (b) Prepare a frequency distribution based on following data. Also draw the frequency polygon for the data. **07**

18.5	21.4	15.9	11.2	12.5	20.8
13.1	25.3	12.6	17.4	23.7	12.6
23.4	18.7	26.2	19.0	10.9	28.9
10.3	20.0	11.8	10.1	15.4	24.3
14.1	22.7	19.5	20.6	16.8	29.3

Group the data into a distribution having classes as 10.0-14.9, 15.0-19.9, 20.0-24.9,.....

Q.5 (a) A random sample of size $n=100$ is taken from a population with $\sigma = 5.1$. Given that sample mean is $\bar{x} = 21.6$, construct a 95% confidence interval for the population mean μ . **07**

(b) Test the null hypothesis $\mu = 0.340$ against the alternate hypothesis $\mu \neq 0.340$ with a level of significance 0.05 given that the sample size is 35 and sample mean is 0.343 and population standard deviation is 0.010. **07**

OR

- Q.5 (a)** A research worker wants to determine the average time it takes a mechanic to rotate the tires of a car, and he wants to be able to assert with 95% confidence that the mean of the sample is off by at most 0.50 minute. If he can presume from past experience that $\sigma = 1.6$ minutes, how large a sample will he have to take? **07**
- (b)** A traveling firm is suspicious of the claim that the average lifetime of certain tires is at least 28000 kms. To check the claim, the firm puts 40 of these tires on its buses and gets a mean lifetime of 27463 kms with a standard deviation 1348 kms. What can it conclude if the probability of a type I error is to be at most 0.01? **07**
