Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code: 711503N
Subject Name: Advanced Solid Mechanics

Date: 02 /02 /2011 **Time: 02.30 pm – 05.00 pm**
Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Drawing neat sketch for the displacement of an element, derive the equation for various strain in Polar Coordinate System. **07**

(b) Give characteristics of Airy's stress function. Derive Airy's stress function for the member subjected to pure bending in Polar Co-ordinate System. **07**

Q.2 **(a)** In an infinitely long and wide plate of width: **2b** is subjected uni-axial stress along length, a circular hole of very small radius: **a** is made at the centre of plate, prove that along the inner circumference of the hole, stress changes the nature. Also, prove that the maximum stress will be three times the axial stress. **07**

(b) Drawing neat sketch for the element subjected to 3-D state of Cartesian stresses along with body forces, derive any one basic differential equation of equilibrium and write the other two equations. Also, write the solutions of stresses in terms of Airy's stress function for any 2-D state of stresses. **07**

OR

(b) Drawing neat sketch for an element subjected to body forces, radial stresses, transverse stresses and shear stresses, derive the basic differential equations of equilibrium in 2-D Polar Coordinate System. **07**

Q.3 **(a)** For a solid circular disk of small uniform thickness, material density: ρ, rotating about the center with angular velocity: ω derive the expressions for radial stress and tangential stress. **07**

(b) For the following state of stresses, find the principal stresses using CARDAN's method and the direction cosines of principal stresses. **07**

Normal stresses: $\sigma_{xx} = 500$ MPa, $\sigma_{yy} = -100$ MPa, $\sigma_{zz} = -50$ MPa, and

Shear stresses: $\tau_{xy} = 90$ MPa, $\tau_{yz} = 45$ MPa, $\tau_{zx} = -60$ MPa,

OR

Q.3 **(a)** Explain soap film bubble analogy and derive the following equation with usual notations: **07**

$\nabla^2 (\phi/2G\theta) = \nabla^2 (sz/p) = -1.$

Comment on usefulness of membrane analogy for solution of complex problems in torsion.

(b) A thick walled cylinder of internal 150 mm ϕ is subjected to an internal and external pressure of 450 MPa and 50 MPa respectively. If the working stress for the material is 425 MPa, determine the thickness of the wall. Also, calculate stresses at mid thickness. **07**

Q.4 **(a)** A point in a strained ductile material, the stresses are acting as σ_x = 600 MPa (T), σ_y = 280 MPa (T) & τ_{xy} = 120 MPa. If yield stress of the material is 150 MPa, check weather material is safe using Tresca shear stress yield theory and/or maximum distortion energy theory. **07**

(b) State generalized Hooke's Law with various relationship for elastic constants (i) modulus of elasticity (ii) Modulus of rigidity (iii) Lami's constant (iv) Bulk modulus (v) Poisson's ratio **07**

OR

Q.4 **(a)** A rectangular column having depth 1:2 times the width is subjected to eccentric load of 1000 kN in a plane bisecting the depth, which produces a moment of 20 kJ at the axis. If the maximum stress induced in the material is 25MPa, calculate the size of column. Use CARDAN's method. **07**

(b) Given the following stress field in MPa **07**

$\sigma_x = 1.38x^3 + 0.0552y$, $\sigma_y = 1.104\ x^2 + 13.8$ & $\tau_{xy} = 2.76z + 2.208y^2$

Calculate strain at a point (4,5,2). Assume E= 210 GPa & Poisson's ratio v = 0.20.

Q.5 **(a)** What is the concept of stability of structures? Give basis of stability of analysis for a slender straight column as well as column initially bent. **07**

(b) Derive equation of buckling load & deformation for the column with both end fixed which produces structural instability. **07**

OR

Q.5 **(a)** A solid circular shaft up of Cast Iron is 2 m long, 25 mm diameter & fixed at one end is subjected to a torque T = 1200 N-m at the free end. Check whether material is safe using maximum normal stress theory and/or Mohr failure criteria. The ultimate tensile stress of CI is 170 MPa & compressive stress is 425 MPa. **07**

(b) Discuss effect of transverse shear on buckling of the beam & derive equation of critical load for the same. **07**

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