

GUJARAT TECHNOLOGICAL UNIVERSITY**M.E Sem-I Regular Examination January / February 2011****Subject code: 712901N****Subject Name: Mathematics for Researcher****Date: 31 /01 /2011****Time: 02.30 pm – 05.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Prove : **07**

$$\int_{x_0}^{x_0 + nh} f(x) dx = \frac{h}{2} \left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right]$$

(b) Define Factorization method **07****Q.2 (a) State and Prove Fourier Integral Theorem.** **07****(b) Develop $\sin \frac{\pi x}{l}$ in half range cosine series in the range $0 \leq x \leq l$** **07****OR****(b) Find the fourier transform of the function e^{-ax^2} , where $a > 0$.** **07****Q.3 (a) The function $y = f(x)$ has a minimum in the interval $0.2 < x < 1.4$. Find the X co-ordinate of the minimum point from the table given below.** **07**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y:	2.10022	1.98730	1.90940	1.86672	1.85973	1.88737	1.95063

(b) Evaluate **04**

$$\int_1^2 e^{-x/2} dx \text{ using four intervals, compare it with the exact value.}$$

(c) Calculate the value of $\int_0^{\frac{\pi}{2}} \sqrt{\sin x} dx$ by using Simpson's $\frac{1}{3}$ rd rule. **03****OR****Q.3 (a) Given that** **07**

X:	1.0	1.1	1.2	1.3	1.4	1.5	1.6
Y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ and $x = 1.6$ **(b) Round off the number 865230 to four significant figures and compute the absolute, relative and percentage error.** **04****(c) Divide 0.9643E21 by 0.7215E - 91** **03****Q.4 (a)** **07**Find the inverse of the matrix using Gauss_Jordan method $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

- (b) Find the dominant eigen value and corresponding eigen vector of the matrix.

07

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \quad \text{starting from } x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

OR

- Q.4 (a) Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 1 & 4 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 2 & 5 & 3 & 2 \end{bmatrix}$ given that if $A_1 = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then $A_1^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ 07

- (b) Apply Jacobi's method to find all the eigen values and the eigen vector of the matrix 07

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

- Q.5 (a) Express Modified Euler's method. 07

- (b) Given $\frac{dy}{dx} = xy$ with $y(1) = 5$. Find the solution correct to three decimal position in the interval $[1, 1.5]$ using step size $h = 0.1$ 07

OR

- Q.5 (a) Solve numerically the equation $y'' + y + 1 = 0$ with boundary condition $y = 0$, when $x = 0$ and $y = 0$ when $x = 1$. 07

- (b) Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by Gauss-Seidel's method. 07
