Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – II • EXAMINATION – SUMMER • 2013

Subject code: 1722309 **Date: 07-06-2013**

Subject Name: Numerical Methods

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find a real root of the equation $x^3 + 4x^2 - 1 = 0$ in the interval $[0, 1]$ by using Bisection method. Perform four iterations. **03**

(ii) Find a real root of the equation $3x = cosx + 1$ correct to three decimal places by using Newton-Raphson Method. Take $x_0 = 0$. **04**

(b) Derive the formula for generating approximations of the root by using False Position Method and hence find a root of the equation **07**

$$xe^x - cosx = 0$$

using False Position Method correct up to $\varepsilon_a < 10\ \%$ in $[0,\ 1]$.

Q.2 **(a)** A total charge Q is uniformly distributed around a ring-shaped conductor with radius a. A charge q is located at a distance x from the center of the ring. The force exerted on the charge by the ring is given by **07**

$$F = \frac{1}{4\pi e_0}\frac{qQx}{(x^2 + a^2)^{\frac{3}{2}}}$$

where $e_0 = 8.85 \times 10^{-12} C^2/(Nm^2)$. Find the distance x where the force is 1.25 N if q and Q are $2 \times 10^{-5} C$ for a ring with radius 0.9 m using Newton-Raphson Method.

(b) What is mathematical modeling? Develop a model to find the velocity v of a freely falling parachutist as a function of time t assuming that the air resistance is linearly proportional to v. How can you solve the problem numerically? **07**

OR

(b) Describe the Gauss Elimination Algorithm. **07**

Q.3 **(a)** (i) Solve the following system of equations by Gauss Elimination Method: **03**

$$x + y + z = 9, \quad 2x - 3y + 4z = 13, \quad 3x + 4y + 5z = 40$$

(ii) Solve the following system of equations using Gauss Seidel Method using initial guess $x_0 = y_0 = z_0 = 0$: **04**

$$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25$$

(b) Use matrix Inversion Method to find the solution of the system: **07**

$$x + y + 2z = 4, x + 2y + 3z = 8, 2x + 3y + z = 12.$$

OR

Q.3 **(a)** Describe in detail the pitfalls of Gauss Elimination Method. Also explain how you will detect a singular system. **07**

(b) The following system of equations was generated by applying the mesh current law to a circuit: **07**

$60I_1 - 40I_2 = 200,$
$40I_1 + 150I_2 - 100I_3 = 0,$
$-100I_2 + 130I_3 = 230$

Solve for I_1, I_2 and I_3.

Q.4 **(a)** (i) The following data was taken from an experiment that measured the current in a wire for various imposed voltages: **05**

V, V	2	3	4	5	7	10
i, A	5.2	7.8	10.7	13	19.3	27.5

On the basis of a linear regression of this data, determine current for a voltage of 3.5 V.

(ii) What is the difference between the two methods of curve fitting, namely, regression and interpolation? **02**

(b) Give the names of three of the nonlinear curves which can be transformed to a linear equation. Describe the procedure to fit such nonlinear curves to the given data. **07**

OR

Q.4 **(a)** Find the equation of a straight line, $y = a_0 + a_1 x$ which best fits with the given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ using least squares method. **07**

(b) (i) Apply Newtonôs backward difference formula to the data below to obtain a polynomial of degree 4 in x: **03**

x	1	2	3	4	5
y	1	-1	1	-1	1

(ii) Use Newtonôs divided difference formula to evaluate $f(8)$ given **04**

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Q.5 **(a)** (i) The following table gives the velocity v of a particle at time t: **03**

t (sec)	0	2	4	6	8	10	12
v (m/sec)	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds.

(ii) Write the algorithm of the Lagrangeôs interpolation method. **04**

(b) Apply Runge-Kutta method of order 4 to find approximate value of y for $x = 0.2$, in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ when $x - 0$. **07**

OR

Q.5 **(a)** A steady state heat balance for a 10-m rod can be represented as **07**

$$\frac{d^2T}{dx^2} - 0.15T = 0$$

with $T(0) = 240$ and $T(10) = 150$. Use the finite-difference approach with $\Delta x = 2\ m$ to solve the given boundary value problem.

(b) (i) Why do we need the methods for numerical integration? **03**

(ii) Describe the algorithm of Euler Method. **04**

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