Seat No.: ________ *Enrolment No.*___________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2014

Subject code: 710401N **Date: 13-06-2014**

Subject Name: Statistical Signal Analysis

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define CDF. State and prove its properties. **07**

(b) (i). Explain Total probability and Baye's theorem **04**

(ii). The probability of a bit error in communication line is 10^{-3}. Find the probability that a block of 1000 bits has five or more errors. **03**

Q.2 **(a)** A company producing electric relays has three manufacturing plants A, B and C producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

(i). If a relay is selected at random from the output of the company, what is the probability that it is defective?

(ii). If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant B? **07**

(b) (i). Let $Y = a\cos(\omega t + \theta)$ where a, ω and t are constants and θ is a uniform random variable in the interval $(0,2\pi)$. The random variable Y results from sampling the amplitude of a sinusoid with random phase θ. Find the expected values of Y and expected value of power of Y. **04**

(ii). Find the variance of the random variable X that is uniformly distributed in the interval [a, b]. **03**

OR

(b) Give answer of following questions.

(i). Discuss statistical independence and uncorrelation of random variables. **04**

(ii). Give the expression for the PDF of a Gaussian random variable and show that Gaussian PDF integrates to one. **03**

Q.3 **(a)** State and explain Markov and Chebyshev's inequalities. **07**

(b) Find the normalization constant c and the marginal PDF's for the following joint PDF :

$f_{X,,Y}(x,y) = ce^{-x}e^{-y}$, $0 \le y \le x \le \infty$

$= 0$, elsewhere

Also, Find $P[X + Y \le 1]$. **07**

OR

Q.3 **(a)** State and prove Central limit theorem. **07**

(b) Let the random variable Y be defined by $Y = aX + b$, Where a is a nonzero constant. Suppose that X has CDF $F_X(x)$, then find $F_Y(y)$ and $f_Y(y)$. **07**

Q.4 **(a)** For random process define cross correlation function and cross power spectral density. Give useful property of cross power spectral density. **07**

(b) The joint CDF for the vector of random variables X = (X,Y) is given by

$$F_{X,Y}(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}), \quad x \geq 0,\ y \geq 0$$
$$= 0, \quad \text{elsewhere}$$

Find the marginal CDF's.

Find the probability of events $A = \{X \leq 1, Y \leq 1\}$, $B = \{X > x, Y > y\}$. **07**

OR

Q.4 (a) What is convergence of Random variable? Explain sure convergence, Almost sure convergence. **07**

(b) A random telegraph signal is passed through a RC low pass filter which has a Transfer function

$$H(f) = \frac{\beta}{\beta + j2\pi f}$$

Where $\beta = 1/RC$ is the time constant of the filter. The Auto correlation function of the Telegraph signal is $R_X(\tau) = 1 - e^{-2\alpha|\tau|}$.

Find the power spectral density and the autocorrelation of the output. **07**

Q.5 (a) (i). Define characteristics function and Moment generating function of random variable. **03**

(ii). For a random process X(t), give the definitions of Mean, Autocorrelation Auto covariance and Correlation coefficient. **04**

(b) Consider a random amplitude sinusoid signal with period T,

$$X(t) = A\cos(2\pi t/T)$$

Is X(t) cyclostationary ? Wide sense cyclostationary ? **07**

OR

Q.5 (a) Give the answer of following questions.

(i). Are the wiener and Poisson processes mean square continuous? **03**

(ii). Does the Wiener process whose autocorrelation function is given by $R_X(t_1,t_2) = \alpha \min(t_1, t_2)$ have a mean square derivative? What is the name of the process obtained by taking derivative of Wiener process? **04**

(b) Explain Mean square convergence and convergence in probability with an example. **07**

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