

GUJARAT TECHNOLOGICAL UNIVERSITY
M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 2710210**Date: 13-01-2015****Subject Name: Numerical Method for Computer Engineering****Time: 02:30 pm - 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

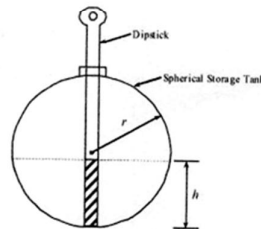
- Q.1 (a)** Define Errors. The solution of a problem is given as 3.436. It is known that the absolute error in the solution < 0.01 . Find the interval within which the exact value must lie. **07**
- (b)** Explain Mathematical modeling with example. What is the role of Mathematical modeling in engineering problem solving? **07**

- Q.2 (a)** To find the inverse of a value a , one can use the equation **07**

$$f(c) = a - \frac{1}{c} = 0 \quad \text{where } c \text{ is the inverse of } a.$$

Use the bisection method of finding roots of equations to find the inverse of $a = 2.5$. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

- (b)** You have a spherical storage tank containing oil. The tank has a diameter of 6 ft. You are asked to calculate the height h to which a dipstick 8 ft long would be wet with oil when immersed in the tank when it contains 6 ft^3 of oil. **07**



The equation that gives the height h of the liquid in the spherical tank for the given volume and radius is given by $f(h) = h^3 - 9h^2 + 3.8197 = 0$

Use the Newton-Raphson method of finding roots of equations to find the height h to which the dipstick is wet with oil. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

OR

- (b)** Obtain the value of expression $(1+x)^2$ by two different methods using 4 digit floating point arithmetic for $x = 0.5129$. Calculate the relative error in two methods. **07**
- Q.3 (a)** Find the unique polynomial $P(x)$ of degree 2 such that: **07**
 $P(1) = 1, P(3) = 27, P(4) = 64$
 Use the Lagrange method of interpolation.
- (b)** Fit a parabola of second degree to the following data **07**
- | | | | | | |
|----|---|-----|-----|-----|-----|
| X: | 0 | 1 | 2 | 3 | 4 |
| Y: | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

OR

Q.3 (a) The upward velocity of a rocket is given at three different times in Table below.

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Time, t (s)	Velocity, v (m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as $v(t) = a_1 t^2 + a_2 t + a_3$, $5 \leq t \leq 12$

The coefficients a_1 , a_2 , and a_3 for the above expression are given by $\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1068 \\ 1772 \\ 2792 \end{bmatrix}$

Find the values of a_1 , a_2 , and a_3 using the Naïve Gauss elimination method. Find the velocity at $t = 6, 7.5, 9, 11$ seconds.

(b) Find the solution to the following system of equations using the Gauss-Seidel method.

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$$12x_1 + 3x_2 - 5x_3 = 1$$

$$x_1 + 5x_2 + 3x_3 = 28$$

$$3x_1 + 7x_2 + 13x_3 = 76$$

Use $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ as the initial guess and conduct six iterations.

Q.4 (a) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using (i) Simpson's one-third rule (ii) Trapezoidal rule

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(b) What do we mean by the 'Best Fit'? What is the meaning of 'Least Square'? Find the best line fit for the following record.

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X: 1.0 1.2 1.4 1.6
Y: 0 0.182 0.336 0.47

Using your 'Best Line Fit' equation find Y when X = 1.8

OR

Q.4 (a) Find an approximate value of $\int_5^8 6x^3 dx$ using Euler's method of solving an ordinary differential equation. Use a step size of $h = 1.5$.

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(b) Evaluate $\log 7$ by Simpson's 3/8 rule. Comment on the accuracy.

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Q.5 (a) A ball at 1200 K is allowed to cool down in air at an ambient temperature of 300 K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

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$$\frac{d\theta}{dt} = -2.2067 \times 10^{-12} (\theta^4 - 81 \times 10^8)$$

where θ is in K and t in seconds. Find the temperature at $t = 480$ seconds using Runge-Kutta 2nd order method. Assume a step size of $h = 240$ seconds.

(b) Consider Table 1 with 14 measurements of the concentration of sodium chlorate produced in a chemical reactor operated at a pH of 7.0.

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Table 1 Chlorate ion concentration in mmol/cm³

12.0	15.0	14.1	15.9	11.5	14.8	11.2	13.7	15.9	12.6	14.3	12.6	12.1	14.8
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Use the data in Table 1 to calculate the

a) mean chlorate concentration b) sample standard deviation c) variance

OR

Q.5 (a) Solve using False position method $x^3 + 2x^2 + 10x - 20 = 0$

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(b) State the various criteria used for curve fitting. Why least square technique is preferred?

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