Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**ME 1st - SEMESTER– I• EXAMINATION – WINTER 2014**

**Subject Code: 2711304 Date:06/01/ 2015**

**Subject Name: Numerical Methods and Statistical Analysis**

**Time: 2:30 to 5:00 pm Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Solve the following system of linear equations using LU Decomposition method by taking $u_{ii} = 1$, where $i = 1, 2, 3$. Also find $A^{-1}$. **07**

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

**(b)** (I) Solve the following tridiagonal system of equations using Thomas algorithm. **04**

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

(II) Show that the following system of linear equations is inconsistent. **03**

$$\begin{bmatrix} 4 & 9 & 3 \\ 2 & 3 & 1 \\ 2 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 7 \end{bmatrix}$$

**Q.2** **(a)** Use four iteration of Rombergôs integration to estimate $ln2 = \int_1^2 \frac{1}{x} dx$. Comment on the accuracy of your result. **07**

**(b)** (I) Evaluate $\int_0^4 te^t \, dt$ using Gaussian Quadrature two point formula and three point formula respectively. **05**

(II) Calculate the infinite norm of $A = \begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}$ **02**

**OR**

**(b)** Obtain the cubic spline approximation for the function defined by the data **07**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | 2 | 33 |

with M(0) = 0, M(2) = 0. Find an estimate of $f(1.5)$.

**Q.3** **(a)** (I) Apply Gauss- Seidel iteration method to solve the equations **05**

$$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25.$$

(II) Round-off the number 465350 to four significant digits and compute relative error. **02**

**(b)** Define: Interpolation and Extrapolation. Find the Lagrange interpolating polynomial that fits the following data values **07**

| $x$ | -1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | -1 | 11 | 31 | 69 |

Also determine the approximate value of $f(1.5)$.

**OR**

**Q.3** **(a)** Using the Choleski decomposition method find solution for the following system of equations **07**

$$x - y + z = 0.5,\ -x + 4y - 2z = 1.5,\ x - 2y + 3z = -1.$$

**(b)** (I) For two events A and B, given that $P(A) = 0.5$, $P(B) = 0.6$, $P(A \cap B) = 0.40$. Find $P((A|B)$, $P(B|A)$ and $P(A \cup B)$. **03**

(II) In a certain assembly plant, three machines, A, B and C make 30%, 45% and 25% respectively, of the products, It is known from past experience that 2%, 3% and 2% of the product made by each machine, respectively are defective. A product is selected randomly and found to be defective. What is the probability that it is produced by machine A. **04**

**Q.4** **(a)** (I) A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20? **02**

(II) Calculate mean and standard deviation from the following data regarding the marks obtained by students in Statistical method **05**

| Marks | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| No of student | 32 | 41 | 57 | 98 | 123 | 83 | 46 | 17 | 3 |

**(b)** (I) A soft-drink machine is regulated so that it discharges an average of 200 ml per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml. **04**

(i) What is the probability that cup will contains more than 224 ml?

(ii) What is the probability that a cup contains between 191 and 209 ml?

(II) A simple random sample of 100 observations was taken from a large population. The sample mean and standard deviation were determined to be 80 and 12 respectively. Compute point estimate, standard error and 95% confidence interval estimate of mean. **03**

**OR**

**Q.4** **(a)** (I) Airline passengers arrive randomly and independently at the passenger screening facility at an international airport. The mean arrival rate is 10 passengers per minute. Compute the probability of **03**

(i) at the most one arrivals in a 1-minute period.

(ii) at least two arrival in a 1-minute period.

(II) The mean height obtained from a random sample of size 100 is 64 inches standard deviation of the distribution of height of the population is 3 inches. Test the statement that mean height population is 67 inches at 1% level of significance. **04**

**(b)** From the data given below about the treatment of 500 patients suffering from a disease. State whether the new treatment is superior to the conventional treatment. **07**

| | No of patients | | |
|---|---|---|---|
| | Favorable | Not favorable | Total |
| New | 280 | 60 | 340 |
| Conventional | 120 | 40 | 160 |
| Total | 400 | 100 | 500 |

(Given for degree of freedom $\nu = 1, \chi^2_{0.05} = 3.84$)

**Q.5** **(a)** (I) The mean life time of a sample of 400 fluorescent light bulbs, produced by a company is found to be 1570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean life time of the bulbs produced by the company is greater than or equal to 1600 hours against the alternatively hypothesis that it is smaller than 1600 hours at 1% level of significance. **04**

(II) The mean and standard deviation of a binomial random variable X are 8 and 2 respectively. Find $P(X = 2)$. **03**

**(b)** Ten specimens of copper wires drawn from a large lot have the following breaking strength (kg. wt) **07**

578 572 570 568 572 578 570 572 596 584

Test the whether the mean breaking strength of the lot may be taken to be 578 kg. wt.t value for degree of freedom=9 at 1% significance level is 3.250

**OR**

**Q.5** **(a)** Find the multiple linear regression equation of $X_1$, $X_2$ and $X_3$ from the data relating to three variables given below: **07**

| | | | | | | |
|---|---|---|---|---|---|---|
| $X_1$ | 4 | 6 | 7 | 9 | 13 | 15 |
| $X_2$ | 15 | 12 | 8 | 6 | 4 | 3 |
| $X_3$ | 30 | 24 | 20 | 14 | 10 | 4 |

**(b)** (I) Obtain the Least square approximation of the form $f(x) = ae^{bx}$ to the data **05**

| $x$ | 0.5 | 1.0 | 2.0 | 2.5 | 3.0 |
|---|---|---|---|---|---|
| $f(x)$ | 0.57 | 1.46 | 5.10 | 7.65 | 9.20 |

(II) Explain the following different type of errors occurring in numerical analysis. (a)Truncation Error (b) Machine epsilon Error **02**

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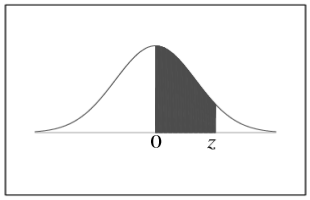
# Standard Normal Distribution Table




| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 |

Gilles Cazelais. Typeset with LaTeX on April 20, 2006.