Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Examination January 2010

Subject code: 710401 **Subject Name: Statistical Signal Analysis**

Date: 06 / 04 / 2010 **Time: 12.00 noon – 02.30 pm**

Total Marks: 60

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** A binary source generates digits 1 and 0 randomly with probabilities P(1) = 0.7 and P(0) =0.3 . If 10 bit are generated. What is the probability that **04**
(i) No 1's will be generated ?
(ii) Eight 1's and two 0's will be generated?
(iii) At least 5 ones will be generated?
(iv) More than 2 zeros will be generated?

(b) A random variable X has pdf **04**

$$f_X(x) = cx(1-x) \quad 0 \le x \le 1$$
$$= 0 \quad \text{elsewhere}$$

(i) Find c (ii) Find $P[1/2 \le X \le 3/4]$
(iii) Find the cdf of X

(c) Show that the random process **04**
$X(t) = A\cos(\omega_c t + \Theta)$
Where Θ is uniformly distributed in the range $(0,2\pi)$,is a wide sense stationary process.

Q.2 **(a)** State and prove the Central Limit Theorem **06**

(b) Find the normalization constant *c* and the marginal pdf's for the following joint pdf. **06**

$$f_{X,Y}(x,y) = ce^{-x}e^{-y} \quad 0 \le y \le x < \infty$$
$$= 0 \quad \text{elsewhere}$$

OR

(b) Show that the *mean square derivative* of a random process X(t) at the point t exists if **06**

$$\frac{\partial^2}{\partial t1 \partial t2} R_x(t1,t2)$$

Exists at the point (t1,t2) =(t,t)

Q.3 **(a)** Find the expression for *mean* and *variance* of **06**
(i) Z = X + Y , X and Y are random variables
(ii) Sum on n *independent identically distributed (iid)* random variable each with a mean μ and variance σ^2

(b) State the Condition for *sure convergence* and *absolute sure convergence* for a sequence or random variables $\{X_n(\zeta)\}$.Check for *convergence* of the following sequences. **06**

(i) $Un(\zeta) = \dfrac{\zeta}{n}$ (ii) $Vn(\zeta) = \zeta(1 - \dfrac{1}{n})$
(iii) $W_n(\zeta) = \zeta e^n$

OR

Q.3 **(a)** Define the Cumulative Distribution Function(cdf) for a random variable and show that $P[a < x \le b] = F_x(b) - F_x(a)$ **04**

	(b)	Give the expression for the pdf of a Gaussian Random variable and its application and show that the Gaussian pdf integrates to 1.	**04**
	(c)	Derive the expressions for *Markov Inequality* and *Chebyshev Inequality*.	**04**

Q.4 **(a)** The joint cdf for the vector of random variables X=(X,Y) Is given by **06**

$F_{X,Y}(x,y) = (1- e^{-\alpha x})(1-e^{-\beta y})$ $\quad x\geq 0, y\geq 0$

$= 0$ $\quad$ elsewhere

Find the probability that events(i) A= $\{X \leq 1, Y\leq 1\}$,
(ii) B=$\{X>x, Y>x\}$, where x >0 ,y >0 , and
(iii) D = $\{1 < X\leq 2, 2<Y\leq 5\}$

(b) Find the pdf of the sum Z = X+Y of two zero mean ,unit variance Gaussian random variables with correlation coefficient ρ = -1/2.(The joint pdf of X and Y is given by **06**

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-(x^2-2\rho xy+y^2)/(2(1-\rho^2)}$$

$-\infty < x, y < \infty$) .Is Z a Gaussian Random variable.

OR

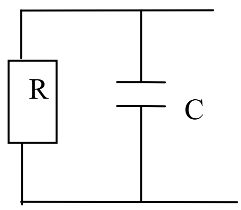
Q.4 **(a)** For a random process X(t) give the definitions of (i) mean (ii) Autocorrelation (iii) Auto covariance (iv) Correlation coefficient) **04**

(b) Does the Weiner Process whose autocorrelation function is given by **04**
$R_X(t_1,t_2) = \alpha \min(t_1,t_2)$
Have a mean square derivative? What is the name of the process obtained by taking the derivative of Wiener Process?

(c) Suppose we observe a process Y(t) which consists of desired signal X(t) plus noise N(t) . Find the cross correlation between the observed signal and the desired signal , assuming that X(t) and N(t) are independent random processes. **04**

Q.5 **(a)** Determine the PSD and mean square value of a random process **06**
$X(t) = A\cos(\omega_c t + \Theta)$

(b) Calculate the thermal noise voltage (rms value) across the RC circuit given in figure below **06**



OR

Q.5 **(a)** A random telegraph signal is passed through a RC lowpass filter which has a Transfer function **06**

$$H(f) = \frac{\beta}{\beta + j2\pi f}$$ where β = 1/RC is the time constant of the filter.

The Auto correlation function of the Telegraph signal is
$R_X(\tau) = 1-e^{-2\alpha|\tau|}$
Find the power spectral density and the autocorrelation of the output.

(b) Consider a random amplitude sinusoid signal with period T **06**
$X(t) = A\cos(2\pi t/T)$
Is X(t) cylclostationary ? Wide sense cyclostationary ?

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