Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 710107N Date: 11-01-2013

Subject Name: Quantum theory & Algorithm Design

Time: 02.30 pm – 05.00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) For the given vectors $|u\rangle = \begin{bmatrix} -1 \\ 7i \\ 2 \end{bmatrix}$ $|v\rangle = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$ **07**

And then compute $7|u\rangle + 2|u\rangle$. **04**

(ii) What is a qubit? What is a superposition state? **03**

(b) Write LU-Decomposition algorithm. Calculate L and U for the matrix **07**

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 6 & 13 & 5 & 19 \\ 2 & 19 & 10 & 23 \\ 4 & 10 & 11 & 31 \end{bmatrix}$$

Q.2 **(a)** The Pauli operators are given by **07**

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

In the $\{|0\rangle, |1\rangle\}$ basis. Find the action of these operators on the basis stated by considering the column vector representation of the $\{|0\rangle, |1\rangle\}$ basis.

(b) Write Kruskal's minimum cost spanning tree algorithm. Prove the theorem that Kruskal's algorithm generates a minimum cost spanning tree for every connected undirected graph G. **07**

OR

(b) Explain and write the procedure for inserting and deleting an element from binary search tree. **07**

Q.3 **(a)** (i) Find $X \otimes Z|\Psi\rangle$, where **07**

$$|\Psi\rangle = \frac{|0\rangle|0\rangle - |1\rangle|1\rangle}{\sqrt{2}}$$ **04**

(ii) Define density operator for a pure state. **03**

(b) (i) Write the pseudo code for Euler tour of the subtree rooted at a node v. **07**

(ii) Obtain a set of optimal Huffman codes for messages(M_1,….,M_7) with relative frequencies(q_1,….,q_7) =(4,5,7,8,10,12,20). Draw the decode tree for this set of codes. **04** **03**

OR

Q.3 **(a)** (i) Does $\rho = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ represent a density operator? If not why not? **07** **04**

(ii) Define density operator for a mixed state. **03**

(b) (i) Write Bellman Ford algorithm. **07**

(ii) Prove the correctness of Dijkstr's algorithm. **04** **03**

Q.4 **(a)** **07**

Suppose $\rho = \begin{bmatrix} \frac{1}{3} & \frac{i}{3} \\ -\frac{i}{3} & \frac{2}{3} \end{bmatrix}$

a. Show that ρ is Hermitian and has positive eigenvalues that satisfy $0 \le \lambda_i \le 1$ and $\sum \lambda_i = 1$.

b. Is this a mixed state?

c. Find $|X\rangle$ for this state.

(b) **07**

(i) Define partial order and total order in relations. **04**

(ii) Explain preorder traversal. **03**

OR

Q.4 **(a)** For the density matrix given by **07**

$$\rho = \frac{1}{5}\begin{bmatrix} 3 & 1-i \\ 1+i & 2 \end{bmatrix}$$

a. Is this a mixed state?

b. Find $|X\rangle, |Y\rangle$ and $|Z\rangle$ for this state.

Q.4 **(b)** **07**

(i) Let A and B be finite sets, and let f:A→B be a function. Show that **04**

a. If f is injective, then |A|≤|B|.

b. If f is surjective, then |A|≥|B|.

(ii) Define post order traversal. **03**

Q.5 **(a)** (i) A quantum system has a density matrix given by **07** **04**

$$\rho = \frac{5}{6}|0\rangle\langle 0| + \frac{5}{6}|1\rangle\langle 1|$$

What is the probability that the system is in the state $|0\rangle$? **03**

(ii) Explain projective measurements.

(b) Consider the following instance of the knapsack problem: n=3,m=20(total weight of knapsack),(p_1,p_2,p_3)=(25,24,15) and (w_1,w_2,w_3)=(18,15,10). Find all the feasible solutions. Out of all which solution yields maximum profit if greedy strategy is applied? Write algorithm for greedy strategies for the knapsack problem. **07**

OR

Q.5 **(a)** (i) What is the use of Bloch vector? Define Bloch vector. **07**

(ii) Consider the state $|\Psi\rangle = \begin{bmatrix} \cos\theta \\ i\sin\theta \end{bmatrix}$ **04** **03**

Is this state normalized?

(b) What is amortized analysis? Show how to apply accounting method of amortized analysis on stack operations and for incrementing a binary counter. **07**

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