Seat No.: ________ Enrolment No.________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – II • EXAMINATION – WINTER • 2013

Subject code: 1722309 **Date: 04-01-2014**

Subject Name: Numerical Methods

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find a real root of the equation $x^3 - 4x - 9 = 0$ in the interval $[2, 3]$ by using Bisection method. Perform four iterations. **03**

(ii) Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to two decimal places by using False Position Method in $[2, 3]$. **04**

(b) Give the geometrical interpretation of Newton-Raphson Method and hence derive its formula. Also find a root of the equation **07**

$$x\, log_{10}^{x} - 1.2 = 0$$

using Newton-Raphson Method correct up to $\varepsilon_a < 0.1\%$. Take $x_0 = 2$.

Q.2 **(a)** In a circuit with a resistor, an inductor, and a capacitor in parallel, the impedance of the system is given by **07**

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

where Z = impedance (Ω) and ω = the angular frequency. Find ω that results in an impedance of 75 Ω with the following parameters:

$$R = 225\ \Omega, C = 0.6 \times 10^{-6} F, L = 0.5\ H \text{ and } \varepsilon_a < 0.1\ \%.$$

Take $\omega_0 = 1$ and $\omega_1 = 1000$ as initial approximations.

(b) Describe different types of errors. **07**

OR

(b) Describe the Gauss Seidel Algorithm. **07**

Q.3 **(a)** (i) Solve the following system of equations by Gauss Jordan Method: **03**

$$2x + 4y + z = 3, \quad 3x + 2y - 2z = -2, \quad x - y + z = 6.$$

(ii) Perform only two iterations of Gauss Seidel Method to solve the following system of equations using $x_0 = y_0 = z_0 = 0$: **04**

$$54x + y + z = 110\ , 2x + 15y + 6z = 72\ , -x + 6y + 27z = 85$$

(b) Use Gauss Jordan Method to find the inverse of the matrix **07**

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

and hence find the solution of the system:

$$x + y + 3z = 1, \quad x + 3y - 3z = 2, \quad -2x - 4y - 4z = 3.$$

OR

Q.3 **(a)** (i) What is partial pivoting? Why do you need it? **03**

(ii) What is an ill-conditioned system? What is scaling and why do you need it? **04**

(b) The following system of equations was generated by applying the mesh current law to a circuit: **07**

$$55I_1 - 25I_3 = -200,$$
$$-37I_2 - 4I_3 = -250,$$
$$-25I_1 - 4I_2 + 29I_3 = 100.$$

Solve for I_1, I_2 and I_3.

Q.4 **(a)** (i) Growth of bacteria (N) in a culture after t hrs. is given in the following table: **05**

t	0	1	2	3	4	5	6
N	32	47	65	92	132	190	275

Fit a curve of the form $N = ab^t$ and estimate N when $t = 7$.

(ii) How will you apply linear regression to fit the curve $y = ax^b$ to the given data? **02**

(b) The population (p) of a small community on the outskirts of a city grows rapidly over a 20-year period: **07**

t	0	5	10	15	20
p	100	200	450	950	2000

Forecast the population 5 years into the future in order to anticipate the demand for power. Employ an exponential model and linear regression to make this prediction.

OR

Q.4 **(a)** Describe the procedure to find the equation of a parabola **07**

$$y = a_0 + a_1 x + a_2 x^2,$$

which best fits with the given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ using least squares method.

(b) (i) Apply Newton's forward difference formula to find $f(22)$ from the data below: **03**

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

(ii) Use Lagrange's interpolation formula to evaluate $f(9)$ given **04**

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Q.5 **(a)** (i) A river is 80 ft. wide. The depth y in feet at a distance x ft. from one bank is given by the following table: **03**

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section. **04**

(ii) Write the algorithm of the Newton's divided difference method.

(b) Apply Runge-Kutta method of order 4 to find approximate value of y for $x = 1$, in steps of 0.5, if $\frac{dy}{dx} = 4e^{0.8x} - 0.5y$, given that $y = 2$ when $x = 0$. **07**

OR

Q.5 **(a)** A steady state heat balance for a 10-m rod can be represented as **07**

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

with $h' = 0.01\ m^{-2}, T_a = 20^\circ C, T(0) = 40$ and $T(10) = 200$. Use the finite-difference approach with $\Delta x = 2\ m$ to solve the given boundary value problem.

(b) (i) What are the differences between Simpson's one-third rule and Simpson's three-eighth rule? **03**

(ii) Describe the algorithm of Heun's Method. **04**

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