Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 710401N **Date: 23-12-2013**

Subject Name: Statistical Signal Analysis

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define Cumulative Distribution Function. List and prove all the properties of CDF. **07**

(b) Let Y=a cos(ωt+φ) where a, ω and t are constants and φ is a uniform random variable in the interval (0, 2π). The random variable Y results from sampling the amplitude of a sinusoid with random phase φ. Find the expected value of Y and expected value of the power of Y, Y^2. **07**

Q.2 **(a)** Consider a random process *X(t)* defined by **07**

$$X(t)=U \cos t + V \sin t, \qquad -\infty < t < \infty$$

where *U* and *V* are independent RV's, each of which assumes the values -2 and 1 with the probabilities 1/3 and 2/3, respectively. Show that *X(t)* is WSS but not strict-sense stationary.

(b) Let *X* and *Y* denote the amplitude of noise signals at two antennas. The random vector *(X,Y)* has the joint pdf **07**

$$f(x,y) = axe^{-ax^2/2} bye^{-by^2/2} \quad x > 0, y > 0, a > 0, b > 0$$

a. Find the joint cdf.
b. Find P[X >Y].
c. Find the marginal pdf's.

OR

(b) Find the mean and variance of the geometric random variable. **07**

Q.3 **(a)** **i)** A communication system accepts a positive voltage V as input and outputs a voltage Y = αV+N, where $\alpha=10^{-2}$ and N is a Gaussian random variable with parameters m = 0 and σ = 2. Find the value of V that gives $P[Y < 0] = 10^{-6}$. **04**

ii) Explain Markov Inequality. **03**

(b) State and prove central limit theorem. **07**

OR

Q.3 **(a)** If X and Y both are independent then show that E[XY] = E[X] E[Y]. **07**

(b) i) A fair coin is tossed 10 times. Find the probability of the occurrence of 5 or 6 heads. **03**

ii) A deck of cards contains 10 red cards numbered 1 to 10 and 10 black cards numbered 1 to 10. How many ways are there of arranging the 20 cards in a row? Suppose we draw the cards at random and lay them in a row, what is the probability that red and black cards alternate? **04**

Q.4 **(a)** Explain the following: Correlation, covariance and correlation coefficient. **07**

(b) Find the characteristic function of the uniform random variable in the interval [a, b]. Find the mean and variance of X by applying the moment theorem. **07**

OR

Q.4 **(a)** Explain probability generating function. **07**

(b) Let the random variable Y be defined by Y = aX + b, where a is a nonzero constant. Suppose that X has cdf $F_X(x)$, then find $F_Y(y)$. Find $f_Y(y)$. **07**

Q.5	**(a)**	Classify random processes and explain each of them.	**07**
	(b)	Consider the random process $X(t) = Y \cos \omega t$, $t \geq 0$ where ω is a constant and Y is a uniform random variable over (0,1). i) Find $E[X(t)]$. ii) Find the autocorrelation function of X(t). iii) Find the autocovariance function of X(t).	**07**

OR

Q.5	**(a)**	Explain strong and weak law of large numbers.	**07**
	(b)	Show that if $\{X(t), t \in T\}$ is a strict-sense stationary random process, then it is also WSS.	**07**

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