

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 714903N

Date: 06-01-2014

Subject Name: Application of Statistical Techniques & Numerical Methods

Time: 10.30 am – 01.00 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** One process of making green gasoline takes biomass in the form of sucrose and converts it into gasoline using catalytic reactions. At one step in a pilot plant process, the output includes carbon chains of length 3. Fifteen runs with same catalyst produced the yields (gal) **07**

5.57 5.76 4.18 4.64 7.02 6.62 6.33 7.24
5.57 7.89 4.67 7.24 6.43 5.59 5.39 .

Treating the yields as a random sample from a normal population, (a) Obtain the maximum likelihood estimates of the mean yield and the variance. (b) Obtain the maximum likelihood estimate of the coefficient of variation σ / μ .

- (b)** The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill slurry is 3.42 grams with a standard deviation of 0.68 gram. Construct a 99% confidence interval for the true mean weight loss of such grinding balls under the stated conditions. **07**

- Q.2 (a)** A trucking firm is suspicious of the claim that the average lifetime of certain tires is at least 28,000 miles. To check the claims, the firm puts 40 of these tires on its trucks and gets a mean lifetime of 27,463 miles with a standard deviation of 1,348 miles. What can it conclude if the probability of a Type I error is to be at most 0.01? **07**

- (b)** A manufacturer of fuses claims that with a 20% overload, the fuses will blow in 12.40 minutes on the average. To test this claim, a sample of 20 of the fuses was subjected to a 20% overload, and the times it took them to blow had a mean of 10.63 minutes and a standard deviation of 2.48 minutes. If it can be assumed that the data constitute a random sample from a normal population, do they tend to support or refute the manufacturer's claim? **07**

OR

- (b)** Solve the differential equation $y'(x) = -y^2$ for $y(2.0)$ using the Milne-Simpson method with the application of modifier to the corrector. The first four points are given under **07**

i	x_i	$y_i = y(x_i)$
0	1.0	1.0000000
1	1.2	0.8333333
2	1.4	0.7142857
3	1.6	0.6250000

- Q.3 (a)** The following are the average weekly losses of worker-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation: **07**

Before: 45 73 46 124 33 57 83 34 26 17

After: 36 60 44 119 35 51 77 29 24 11

Use the 0.05 level of significance to test whether the safety program is effective.

- (b)** After a large number of assays of the gold content in rocks collected from an open pit mine, a mining engineer postulates that the natural log of the gold content (oz/st gold) follows a normal distribution with mean -4.6 and variance 1.21. Under this distribution, would it be unusual to get 0.0015 oz/st gold or less in an assay? **07**

OR

- Q.3 (a)** Factorize the matrix **07**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$

Using Cholesky's algorithm.

- (b)** Given the equations **07**

$$\frac{dy_1}{dx} = x + y_1 + y_2; \quad y_1(0) = 1$$

$$\frac{dy_2}{dx} = 1 + y_1 + y_2; \quad y_2(0) = -1$$

Estimate the values of $y_1(0.1)$ & $y_2(0.1)$ using Heun's method.

- Q.4 (a)** The following are measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine: **07**

Air velocity (cm/s) X	Evaporation coefficient (mm ² /s) Y
20	0.18
60	0.37
100	0.35
140	0.78
180	0.56
220	0.75
260	1.18
300	1.36
340	1.17
380	1.65

Fit a straight line to these data by the method of least squares, and use it to estimate the evaporation coefficient of a droplet when the air velocity is 190 cm/s.

- (b) The actual amount of instant coffee that a filling machine puts into “4-ounce” jars may be looked upon as a random variable having a normal distribution with $\sigma = 0.04$ ounce. If only 2% of the jars are to contain less than 4 ounces, what should be the mean fill of these jars? **07**

OR

- Q.4 (a)** Evaluate the integral of $f(x) = 1 + e^{-x} \sin 4x$ over the interval $[0, 1]$ using exactly five functional evaluations. **07**

- Q.4 (b)** Compute Romberg estimate R_{22} for $\int_1^2 \frac{1}{x} dx$. **07**

- Q.5 (a)** From the following results obtain the two regression equations and estimate the yield of crops when the rainfall is 29cms and the rainfall when the yield is 600kg. **07**

Mean	Y (yield in kgs)	(Rainfall in cms)
	508.4	26.7
S.D.	36.8	4.6

Coefficient of correlation between yield and rainfall is 0.52.

- (b) Evaluate $\int_1^3 \sin x dx$ using Gauss quadrature of five points. Compare the result with analytic value. **07**

OR

- Q.5 (a)** Find the value of by $\int_4^{5.2} (\log_e x) dx$ Weddle's rule. **07**

- (b) Using Runge-Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$, $\frac{dy}{dx} = xy + y^2, y(0) = 1$ given. Continue the solution at $x = 0.4$ using Milne's method **07**
