Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – III • EXAMINATION – WINTER • 2013

Subject code: 730403 Date: 28-11-2013

Subject Name: Optimization Techniques

Time: 10.30 am – 01.00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the maxima and minima, if any, of the function **03**

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5.$$

(ii) Find the extreme points of the function **04**

$$f(x_1, x_2) = x_1^3 + x_2^3 + x_1^2 + 2x_2^2 + 10.$$

(b) Find the dimensions of a rectangular box of volume $V = 1000\ in^3$ for which the total length of the 12 edges is a minimum using the Lagrange's method of multipliers (No need to check the sufficiency conditions). Find the change in the dimensions of the box when the volume is changed to $1200\ in^3$ by using the value of λ^* found. **07**

Q.2 **(a)** Consider the following problem: **07**

Minimize $f(X) = (x_1 - 2)^2 + (x_2 - 1)^2$

subject to $2 \geq x_1 + x_2,\ \ x_2 \geq x_1^2,$

Determine whether the Kuhn-Tucker conditions are satisfied at the following points:

$$X_1 = [1.5, 0.5]^T, \qquad X_2 = [1, 1]^T, \qquad X_3 = [2, 0]^T.$$

(b) Use the graphical method to minimize $f = -x_1 + 2x_2$ **07**

subject to the constraints

$$-x_1 + 3x_2 \leq 10, \quad x_1 + x_2 \leq 6,$$
$$x_1 - x_2 \leq 2, \qquad x_1, x_2 \geq 0.$$

OR

(b) Use Revised Simplex Method to maximize $f = 3x_1 + 2x_2 + 5x_3$ **07**

subject to $x_1 + 2x_2 + x_3 \leq 430,\ 3x_1 + 2x_3 \leq 460,$

$$x_1 + 4x_2 \leq 420, \quad x_1 \geq 0,\ x_2 \geq 0, x_3 \geq 0.$$

Q.3 **(a)** Use Fibonacci Method with $n = 6$ to maximize the function **07**

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$$

in the interval $[0, 3]$.

(b) Use Secant Method to find the minimum of $f = x^3 - 4x + 9$ with an initial step size of $t_0 = 0.2, \lambda_1 = 0.0,$ and $\varepsilon = 0.01$. **07**

OR

Q.3 **(a)** Use Dichotomous Search Method to find the minimum of $f = x^2 - 1.6x + 4$ in the interval $(0.0, 1.00)$ to within 10% of the exact value. Take $\delta = 0.001$. **07**

(b) Use Newton's Method to minimize the function **07**

$$f(\lambda) = \frac{\lambda}{log\lambda}$$

with $\lambda_0 = 2$. Iterate until $|f'(\lambda)| < 0.01$.

Q.4 **(a)** Perform three iterations of Interval Halving Method to minimize the function $f(\lambda) = 3\lambda^5 - 5\lambda^3 - 30\lambda + 20$ in the interval $(0, 2)$. **07**

(b) What are pattern directions? Draw the flow chart of Powell's method. **07**

OR

Q.4 **(a)** Use Golden Section Method to minimize the function **07**

$$f(\lambda) = 3\lambda^5 - 10\lambda^3 - 45\lambda + 1$$

with $n = 6$ in the interval $(0, 3)$.

(b) Write the algorithm of Univariate Method. **07**

Q.5 **(a)** Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ using Hooke and Jeeves's Method. Take $\varepsilon = 0.1$ and $\Delta x_1 = \Delta x_2 = 0.8$. Perform only two iterations. **07**

(b) Find the dimensions of a rectangular prism type box that has the largest volume when the sum of its length, width and height is limited to a maximum value of 60 $in.$ and its length is restricted to a maximum value of 36 $in.$ **07**

OR

Q.5 **(a)** Use Fletcher-Reeves method to minimize **07**

$$f(x_1, x_2) = 3x_1^2 - 5x_1x_2 + 4x_2^2 - 2x_1 + 3x_2$$

from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.

(b) Describe the basic approach of the penalty function method. Also write the algorithm of Exterior Penalty function Method. **07**

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