Seat No.: ________ *Enrolment No.___________*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – III • EXAMINATION – WINTER • 2013

Subject code: 735206 **Date: 28-11-2013**

Subject Name: Digital Signal Processing

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** For each of the following discrete-time systems, where y[n] and x[n] are, respectively, the output and input sequences, determine whether or not the system is (i) linear (ii)causal (iii) stable and (iv) shift-invariant: **07**

I. $y[n] = n^2 . x[n]$
II. $y[n] = x^4[n]$
III. $y[n] = x[n-5]$

(b) Let $X(e^{j\omega})$ denote the DTFT of a real sequence x[n]. Express the inverse DTFT of y[n] for the below given $Y(e^{j\omega})$ in terms of x[n] **07**

I. $Y(e^{j\omega}) = \frac{1}{2}\left\{X\left(e^{j\omega/2}\right) + X\left(e^{-j\omega/2}\right)\right\}$
II. $Y(e^{j\omega}) = X\left(e^{j3\omega}\right)$

Q.2 **(a)** Let x[n] = $0 \le n \le N-1$, be an even length sequence with an N-point DFT X[k], $0 \le k \le N-1$. Determine the N-point DFTs of the following length – N sequences in terms of X[k]: **07**

I. y1[n] = x[n] – x[N-1];
II. y2[n] = x[n] – x[n-(N/2)]

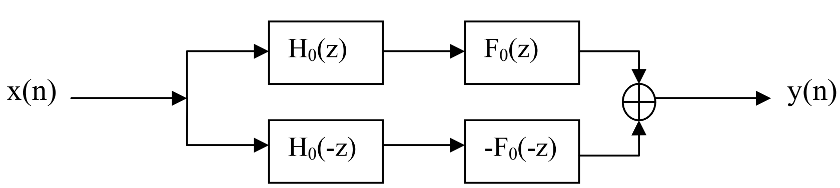
(b) Let G[k] and H[k] denote the 7-point DFTs of the two length-7 sequences g[n] and h[n], respectively. If G[k] = {1+j2, -2+j3, -1-j2, 0, 8+j4, -3+j, 2+j5} and $h[n] = g[(n-3)_7]$, determine the H[k] without determining the DFT. **07**

OR

(b) Determine the Z-transform of the following sequence and its respective ROC. Assume |β| > |α|. Show their pole-zero plots and indicate clearly the ROC in the plot. **07**

I. $x1[n] = (\alpha^n . u[-n-1]) + (\beta^n . u[n])$where u[n] is a unit step function

Q.3 **(a)** Consider the discrete-time system shown below. For $H_0(z) = 1+\alpha.z^{-1}$, find a suitable $F_0(z)$ so that the output is a delayed and scaled replica of the input. **07**



07

(b) An IIR system function is given below

$$\frac{1-z^{-1}}{1-k.z^{-1}}$$

Determine the magnitude response of the above transfer function and show that it has high-pass response. Scale the transfer function so that it has 0-dB gain at $\omega = \pi$. Sketch the magnitude responses for k = 0.95, 0.9 and -0.5 respectively.

OR

Q.3 **(a)** Show that the phase delay $\tau_p(\omega) = -\theta(\omega)/\omega$ of the first-order allpass transfer function $A_1(z) = \dfrac{d_1 + z^{-1}}{1 + d_1 z^{-1}}$ is given by $\tau_p(\omega) \cong (1-d_1)/(1+d_1)$ **07**

(b) A causal stable LTI discrete-time-system is characterized by an impulse response $h_1(n) = -\delta[n] + \frac{5}{3}.(0.5)^n.u(n) + \frac{1}{12}(-0.25)^n.u(n)$. **07**

Determine the impulse response $h_2(n)$ of its inverse system which is causal and stable.

Q.4 **(a)** Consider the below given two analog transfer functions $H_1(s)$ and $H_2(s)$ representing bandpass and bandstop systems respectively **07**

$$H_1(s) = \frac{bs}{s^2 + bs + \Omega_0^2} \quad b > 0 \text{ and}$$

$$H_2(s) = \frac{s^2 + \Omega_0^2}{s^2 + bs + \Omega_0^2} \quad b > 0$$

If $H_1(s)$ and $H_2(s)$ can be expressed in the form

$$H_1(s) = \frac{1}{2}[A_1(s) + A_2(s)] \; ; \; H_2(s) = \frac{1}{2}[A_1(s) - A_2(s)];$$

Where $A_1(s)$ and $A_2(s)$ are stable analog all-pass transfer functions. Determine $A_1(s)$ and $A_2(s)$ functions.

(b) A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 250Hz, 450 Hz, 1.0kHz, 2.75kHz and 4.05kHz. The signal $x_a(t)$is sampled at a 1.5kHz rate and 3.0kHz and find the frequency component present in the reconstructed signals and justify the answer. **07**

OR

Q.4 **(a)** Discuss in detail about the sampling a bandlimited signal in time domain and its effects in the frequency domain. **07**

Explain in detail about the relation between Ω (= 2πf) in radian/sec and ω in radian/sample.

State and explain Niquist Theorem and Niquist Criterion.

(b) Explain in detail about any two Analog to Digital Conversion systems and any two Digital to Analog Conversion systems. **07**

Q.5 **(a)** The transfer function of a second order analog filter with a passband edge of 0.16Hz and a passband ripple of 1dB is given by **07**

$$H_{LP}(s) = \frac{0.056(s^2 + 17.95)}{s^2 + 1.06s + 1.13}$$

By applying appropriate spectral transformation relation, determine the transfer function $H_{BP}(s)$ of an analog bandpass filter with a centre frequency at 3Hz and a bandwidth of 0.5Hz.

(b) Let $H_{LP}(z)$ denote the transfer function of a real-coefficient lowpass filter with a passband edge at ω_p and stopband edge at ω_s, passband ripple of δ_p and stopband ripple of δ_s. Consider a cascade of two identical filters with a transfer function of H(z). What are the passband and stopband ripples of the cascade at ω_p and ω_s, respectively. Generalize the results for a cascade of M identical sections **07**

Sketch the magnitude response of the transfer function $H_{LP}(-z)$ *and* $H_{LP}\left(e^{j\omega_0} z\right)$ for $0 \le \omega \le \pi$

OR

Q.5 **(a)** Evaluate the Frequency Spectra of Hamming, Hann and Blackman Window functions of length (2M+1). Plot them. **07**

Compare the their mainlobe widths with the mainlobe width of same length Rectangular Window

(b) Let $\left|H_d\left(e^{j\omega}\right)\right|$ denote the desired magnitude response of a real linear-phase FIR filter of length M. **07**

i. For M odd, show that the DFT samples H[k] needed for a frequency sampling – based design are given by

$$H[k] = \begin{cases} \left|H_d\left(e^{j2\pi k/M}\right)\right| e^{-j2\pi k(M-1)/2M}, & k = 0,1,2,....\frac{M-1}{2} \\ \left|H_d\left(e^{j2\pi k/M}\right)\right| e^{j2\pi k(M-k)(M-1)/2M}, & k = ,\frac{M+1}{2},2,....M-1 \end{cases}$$

ii. For M odd, show that the DFT samples H[k] needed for a frequency sampling – based design are given by

$$H[k] = \begin{cases} \left|H_d\left(e^{j2\pi k/M}\right)\right| e^{-j2\pi k(M-1)/2M}, & k = 0,1,2,....\frac{M}{2}-1 \\ 0, & k = \frac{M}{2} \\ \left|H_d\left(e^{j2\pi k/M}\right)\right| e^{j2\pi k(M-k)(M-1)/2M}, & k = \frac{M}{2}+1,2,....M-1 \end{cases}$$

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