Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**M. E. - SEMESTER – II • EXAMINATION – WINTER • 2014**

**Subject code: 1720110 Date: 08-12-2014**

**Subject Name: Numerical Methods**

**Time: 02:30 pm - 05:00 pm Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Define absolute and percentage errors. Let the exact value $= 20/3$ and its approximate value $= 6.666$. Find the relative and percentage errors. Also find the number of significant digits. **07**

**(b)** Use Newton's divided difference formula to find $f(x)$ from the following data: **07**

| $x$: | 0 | 2 | 3 | 4 | 6 | 7 |
|---|---|---|---|---|---|---|
| $f(x)$: | 0 | 8 | 0 | -72 | 0 | 1008 |

**Q.2** **(a)** Find the real root of $x\tan x + 1 = 0$ using Bisection method correct up to the three decimal places. **07**

**(b)** Define different types of errors with suitable illustration. **07**

**OR**

**(b)** Explain convergence criteria for Gauss- Seidal method. Use it to solve the system $30x - 2y + 3z = 75$ , $2x + 2y + 18z = 30$ , $x + 17y - 2z = 48$ correct up to four decimal places. **07**

**Q.3** **(a)** Derive Simpson $1/3^{rd}$ rule. Estimate $\int_{0.5}^{1.3} e^{x^2} dx$ using Simpson rule with i) four strips ii) eight strips. **07**

**(b)** Using improved Euler's method, solve $y' = -2xy^2$ with initial condition $y(0) = 1$ and compute $y(1)$ taking step size $0.2$. **07**

**OR**

**Q.3** **(a)** Find the value of $y$ at $x = 21$ and $x = 28$ from the following data: **07**

| $x$: | 20 | 23 | 26 | 29 |
|---|---|---|---|---|
| $y$: | 0.3420 | 0.3907 | 0.4384 | 0.4848 |

**(b)** Fit a straight line to the following data: **07**

| $x$: | 100 | 120 | 140 | 160 | 180 | 200 |
|---|---|---|---|---|---|---|
| $y$: | 0.45 | 0.55 | 0.60 | 0.70 | 0.80 | 0.85 |

**Q.4** **(a)** Explain Newton-Rapshon method graphically. Use it to find the real root of $xe^x - 2 = 0$ correct up to three decimal places. **07**

**(b)** Define Numerical integration. The velocity $v$ of a particle at distance $s$ from a point on its linear path is given in the following data: **07**

| $s(m)$ | 0 | 2.5 | 5.0 | 7.5 | 10.0 | 12.5 | 15.0 | 17.5 | 20.0 |
|---|---|---|---|---|---|---|---|---|---|
| $v(m/\sec)$ | 16 | 19 | 21 | 22 | 20 | 17 | 13 | 11 | 9 |

**OR**

**Q.4** **(a)** Use Runge- Kutta method of fourth order to find $y(1.1)$, given that **07**

$\frac{dy}{dx} = x - y,\ y(1) = 1$ with $h = 0.05$.

**(b)** **07**

Find the inverse of $A = \begin{bmatrix} 2 & 4 & -6 \\ 1 & 5 & 3 \\ 1 & 3 & 2 \end{bmatrix}$ and also solve the system $AX = B$, where

$B = \begin{bmatrix} -4 \\ 10 \\ 5 \end{bmatrix}$.

**Q.5** **(a)** Given that $y(1) = 1.3$ and $y' = 3x + y$. Use second order Runge- Kutta method to approximate $y$ when $x = 1.2$ with $h = 0.1$ **07**

**(b)** Fit cubic spline for following data: **07**

| $x$: | $x_0 = 1$ | $x_1 = 2$ | $x_2 = 3$ | $x_3 = 4$ |
|---|---|---|---|---|
| $y$: | $y_0 = 1$ | $y_1 = 2$ | $y_2 = 5$ | $y_3 = 11$ |

**OR**

**Q.5** **(a)** Define Interpolation. Find the Lagrangeôs interpolating polynomial from the following data: **07**

| $x$: | 0 | 1 | 4 | 5 |
|---|---|---|---|---|
| $f(x)$: | 1 | 3 | 24 | 39 |

Hence find $f(0.7)$ .

**(b)** Discuss False - Position method graphically. Use it to find real root of **07**

$x^3 - 5x + 1 = 0$ correct up to four decimal places.

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